Fundamentals of Engineering Exam Review Series

Mathematics

Prof. Meredith Metzger
Department of Mechanical Engineering
University of Utah

Overview

- 110 multiple choice questions total
- 5 hrs 20 min to answer questions
- slightly less than 3 minutes per question

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- 5 hrs 20 min to answer questions
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Discipline	Number of math questions	% of test
Mechanical	6-9	5.5% - 8%
Electrical & Computer	11-17	10% - 15.5%
Civil	7-11	6% - 10%
Chemical	8-12	7% - 11%
Other	12-18	11% - 16%

Mathematics Content

Discipline	Algebra & Trigonometry	Analytic Geometry	Calculus	Linear Algebra	Vector Analysis	Differential Equations	Numerical Methods	Complex Numbers	Discrete Mathematics	Roots of Equations
Mechanical		✔	✔	✔	✔	✔	✔			
Electrical & Computer	✔	✔	✔	✔	✔	✔		✔	✔	
Civil		✔	✔		✔					✔
Chemical		✔	✔			✔				✔
Other	✔	✔	✔	✔		✔	✔			

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Mechanical		✔	✔	✔	✔	✔	✔			
Electrical & Computer	✔	✔	✔	✔	✔	✔		✔	✔	
Civil		✔	✔		✔					✔
Chemical		✔	✔			✔				✔
Other	✔	✔	✔	✔		✔	✔			

Permitted Calculators

- Casio FX-115 models
- HP 33 models
- HP 35 models
- TI-30x models
- TI-36x models

Outline

I. Analytic Geometry

II. Algebra

III. Trigonometry

IV. Calculus

V. Differential Equations

VI. Linear Algebra and Vectors

Analytic Geometry

- Equations and Curves
- Perimeter, Area, and Volume
- Conic Sections
  - Parabola
  - Hyperbola
  - Ellipse
  - Circle

Straight Line (pg. 18)

STRAIGHT LINE

The general form of the equation is

$$Ax + By + C = 0$$

The standard form of the equation is

$$y = mx + b,$$

which is also known as the *slope-intercept* form.

The *point-slope* form is $y - y_1 = m(x - x_1)$

Given two points: slope, $m = (y_2 - y_1)/(x_2 - x_1)$

The angle between lines with slopes m_1 and m_2 is

$$\alpha = \arctan\left[(m_2 - m_1)/(1 + m_2 \cdot m_1)\right]$$

Two lines are perpendicular if $m_1 = -1/m_2$

The distance between two points is

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Straight Line

A line goes through the point $(4, -6)$ and is perpendicular to the line $y = 4x + 10$. What is the equation of the line?

(A) $y = -\frac{1}{4}x - 20$

(B) $y = -\frac{1}{4}x - 5$

(C) $y = \frac{1}{5}x + 5$

(D) $y = \frac{1}{4}x + 5$

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(D) $y = \frac{1}{4}x + 5$

Straight Line

What is the general form of the equation for a line whose x-intercept is 4 and y-intercept is -6?

(A) $2x - 3y - 18 = 0$

(B) $2x + 3y + 18 = 0$

(C) $3x - 2y - 12 = 0$

(D) $3x + 2y + 12 = 0$

Straight Line

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(C) $3x - 2y - 12 = 0$

(D) $3x + 2y + 12 = 0$

Straight Line

The angle between the line $y = -7x + 12$ and the line $y = 3x$ is most nearly

(A) $22°$

(B) $27°$

(C) $33°$

(D) $37°$

Straight Line

The angle between the line $y = -7x + 12$ and the line $y = 3x$ is most nearly

(A) 22°

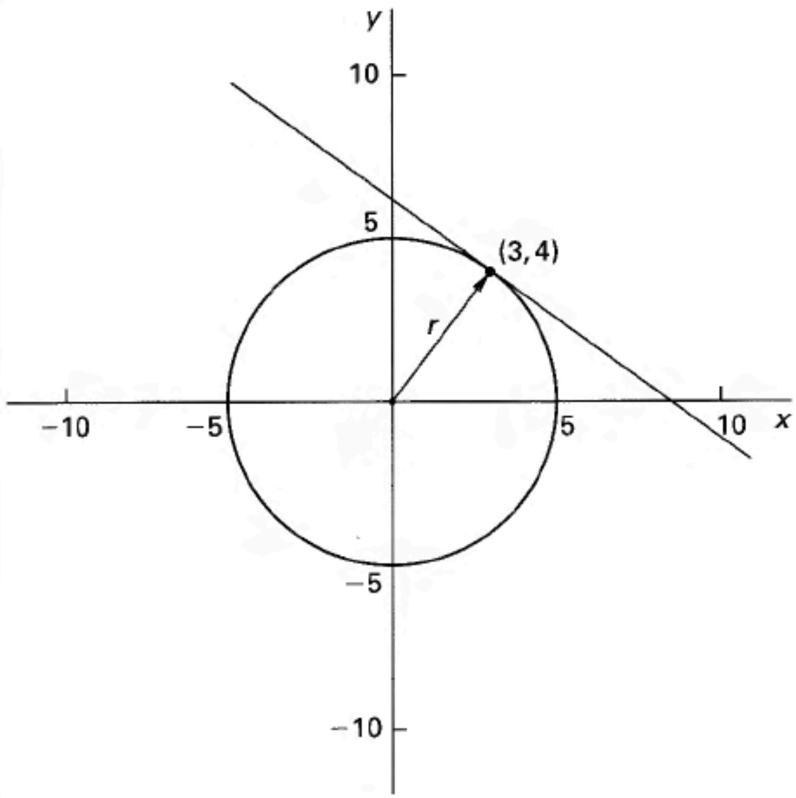
(B) 27°

(C) 33°

(D) 37°

Tangent Line to Circle

A circle with a radius of 5 is centered at the origin.



What is the standard form of the equation of the line tangent to this circle at the point (3, 4)?

(A) $x = \frac{-4}{3}y - \frac{25}{4}$

(B) $y = \frac{3}{4}x + \frac{25}{4}$

(C) $y = \frac{-3}{4}x + \frac{9}{4}$

(D) $y = \frac{-3}{4}x + \frac{25}{4}$

Tangent Line to Circle

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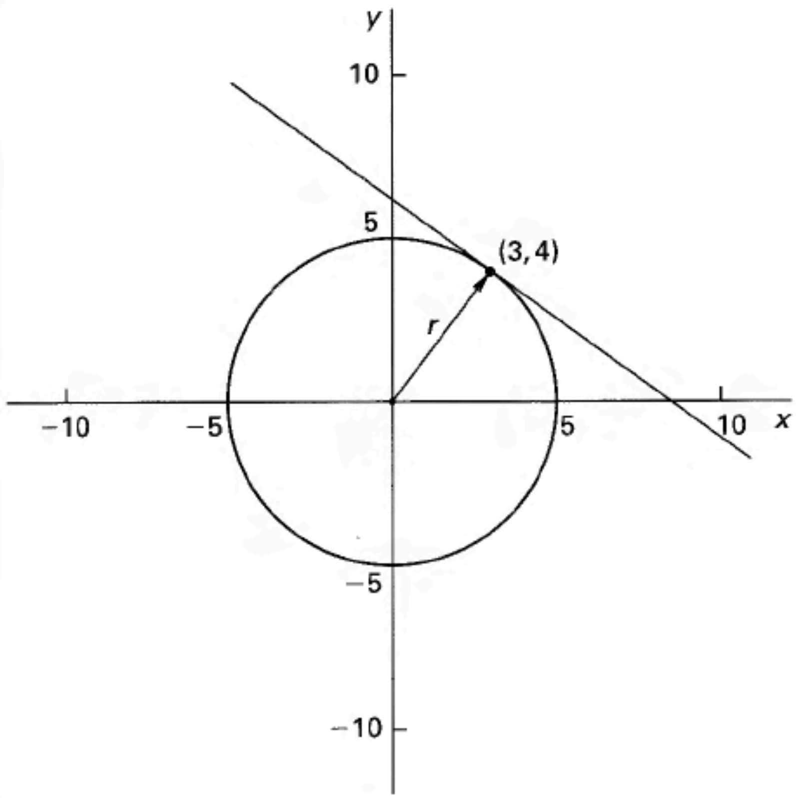

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(A) $x = \frac{-4}{3}y - \frac{25}{4}$

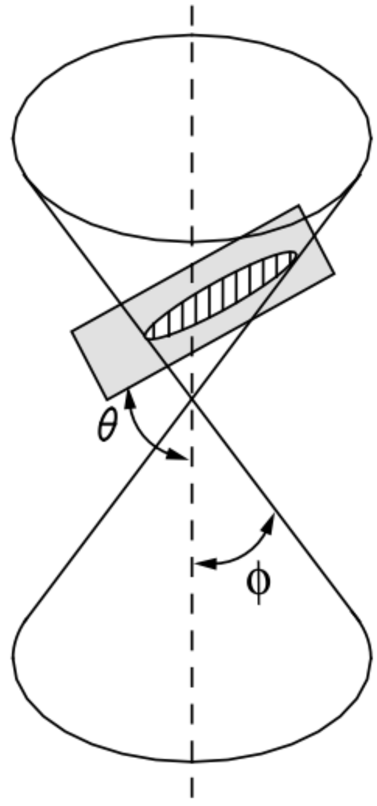
(B) $y = \frac{3}{4}x + \frac{25}{4}$

(C) $y = \frac{-3}{4}x + \frac{9}{4}$

(D) $y = \frac{-3}{4}x + \frac{25}{4}$

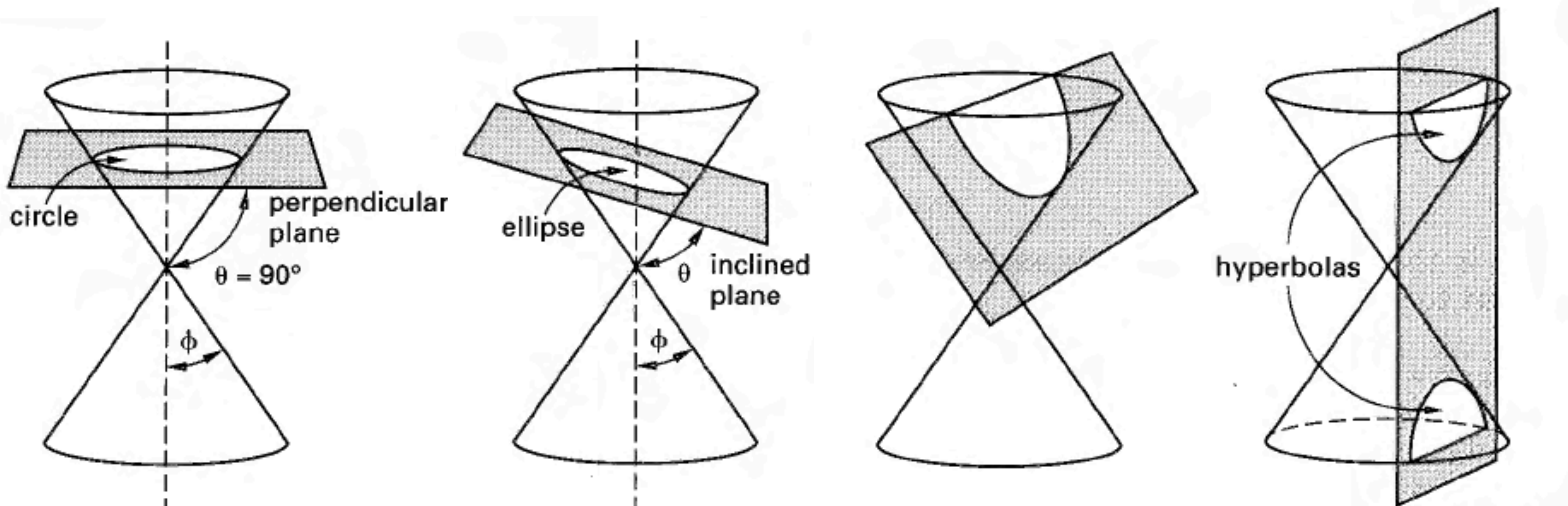
Conic Sections (pgs. 22-23)

Writing equations for various conic sections



e = eccentricity = $\cos\theta/(\cos\phi)$

Conic Sections



(a) circle (θ = 90°) $e = 0$

(b) ellipse (ϕ < θ < 90°) $0 < e < 1$

(c) parabola (θ = ϕ) $e = 1$

(d) hyperbolas (0 ≤ θ < ϕ) $e > 1$

Conic Sections (pgs. 22-23)

Conic Section Equation

The general form of the conic section equation is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where not both A and C are zero.

If $B^2 - 4AC < 0$, an *ellipse* is defined.

If $B^2 - 4AC > 0$, a *hyperbola* is defined.

If $B^2 - 4AC = 0$, the conic is a *parabola*.

If $A = C$ and $B = 0$, a *circle* is defined.

If $A = B = C = 0$, a *straight line* is defined.

$$x^2 + y^2 + 2ax + 2by + c = 0$$

is the normal form of the conic section equation, if that conic section has a principal axis parallel to a coordinate axis.

$h = -a;\ k = -b$

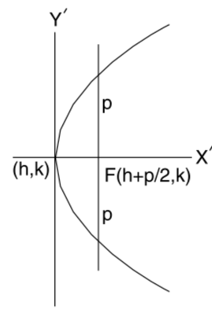
$r = \sqrt{a^2 + b^2 - c}$

If $a^2 + b^2 - c$ is positive, a *circle*, center $(-a, -b)$.

If $a^2 + b^2 - c$ equals zero, a *point* at $(-a, -b)$.

If $a^2 + b^2 - c$ is negative, locus is *imaginary*.

Case 1. Parabola $e = 1$:

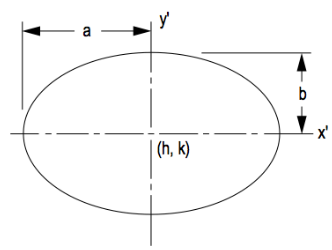
- 

$(y - k)^2 = 2p(x - h)$; Center at (h, k)

is the standard form of the equation. When $h = k = 0$,

Focus: $(p/2, 0)$; Directrix: $x = -p/2$

Case 2. Ellipse $e < 1$:



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1;\ \text{Center at } (h, k)$$

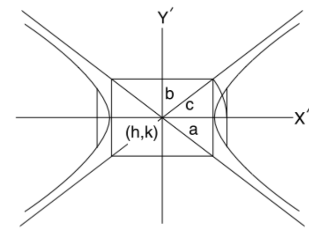
is the standard form of the equation. When $h = k = 0$,

Eccentricity: $e = \sqrt{1 - (b^2/a^2)} = c/a$

$b = a\sqrt{1 - e^2}$;

Focus: $(\pm ae, 0)$; Directrix: $x = \pm a/e$

Case 3. Hyperbola $e > 1$:

- 

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1;\ \text{Center at } (h, k)$$

is the standard form of the equation. When $h = k = 0$,

Eccentricity: $e = \sqrt{1 + (b^2/a^2)} = c/a$

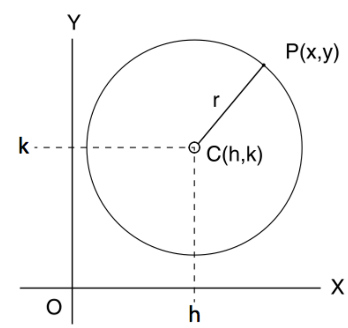
$b = a\sqrt{e^2 - 1}$;

Focus: $(\pm ae, 0)$; Directrix: $x = \pm a/e$

Case 4. Circle $e = 0$:

$(x - h)^2 + (y - k)^2 = r^2$; Center at (h, k) is the standard form of the equation with radius

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

- 

Conic Sections

What kind of conic section is described by the following equation?

$$4x^2 - y^2 + 8x + 4y = 15$$

(A) circle

(B) ellipse

(C) parabola

(D) hyperbola

Conic Sections

What kind of conic section is described by the following equation?

$$4x^2 - y^2 + 8x + 4y = 15$$

(A) circle

(B) ellipse

(C) parabola

(D) hyperbola

Conic Sections

What is the equation of a parabola with a vertex at $(4, 8)$ and a directrix at $y = 5$?

(A) $(x-8)^2 = 12(y-4)$

(B) $(x-4)^2 = 12(y-8)$

(C) $(x-4)^2 = 6(y-8)$

(D) $(y-8)^2 = 12(x-4)$

Conic Sections

What is the equation of a parabola with a vertex at $(4, 8)$ and a directrix at $y = 5$?

(A) $(x - 8)^2 = 12(y - 4)$

(B) $(x - 4)^2 = 12(y - 8)$

(C) $(x - 4)^2 = 6(y - 8)$

(D) $(y - 8)^2 = 12(x - 4)$

Conic Sections

What is the equation of the ellipse with center at (0,0) that passes through the points (2,0), (0,3), and (−2,0)?

(A) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

(B) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(C) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(D) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Conic Sections

What is the equation of the ellipse with center at $(0,0)$ that passes through the points $(2,0)$, $(0,3)$, and $(-2,0)$?

(A) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

(B) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(C) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(D) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Conic Sections

What is the equation of the circle passing through the points $(0,0)$, $(0,4)$, and $(-4,0)$?

(A) $(x-2)^2+(y-2)^2=\sqrt{8}$

(B) $(x-2)^2+(y-2)^2=8$

(C) $(x+2)^2+(y-2)^2=8$

(D) $(x+2)^2+(y+2)^2=\sqrt{8}$

Conic Sections

What is the equation of the circle passing through the points $(0,0)$, $(0,4)$, and $(-4,0)$?

(A) $(x-2)^2 + (y-2)^2 = \sqrt{8}$

(B) $(x-2)^2 + (y-2)^2 = 8$

(C) $(x+2)^2 + (y-2)^2 = 8$

(D) $(x+2)^2 + (y+2)^2 = \sqrt{8}$

Quadratic Surface (pg. 18) & Tangent Line to Circle (pg. 23)

QUADRIC SURFACE (SPHERE)

The standard form of the equation is

$$(x - h)^2 + (y - k)^2 + (z - m)^2 = r^2$$

with center at (h, k, m).

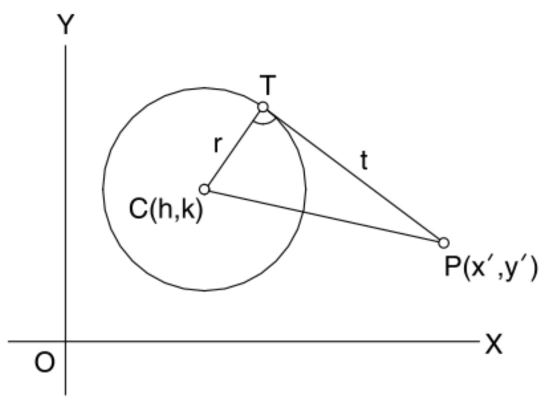
In a three-dimensional space, the distance between two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Length of the tangent line from a point on a circle to a point (x', y'):

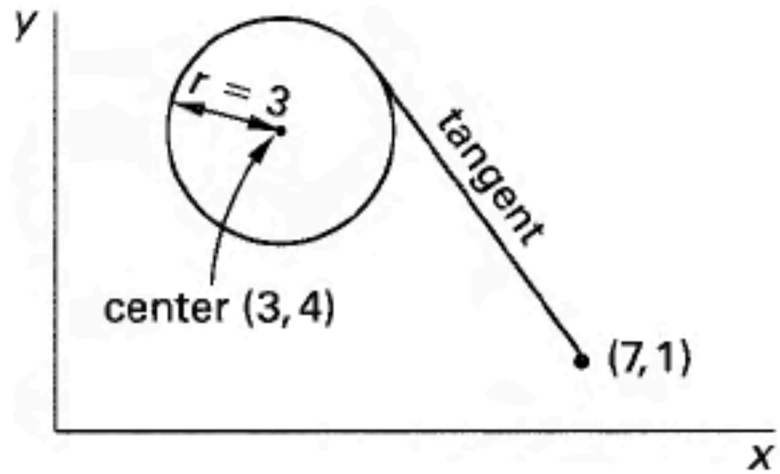
$$t^2 = (x' - h)^2 + (y' - k)^2 - r^2$$

-



Tangent Line to Circle

What is the length of the line tangent from point $(7, 1)$ to the circle shown?



(A) 3

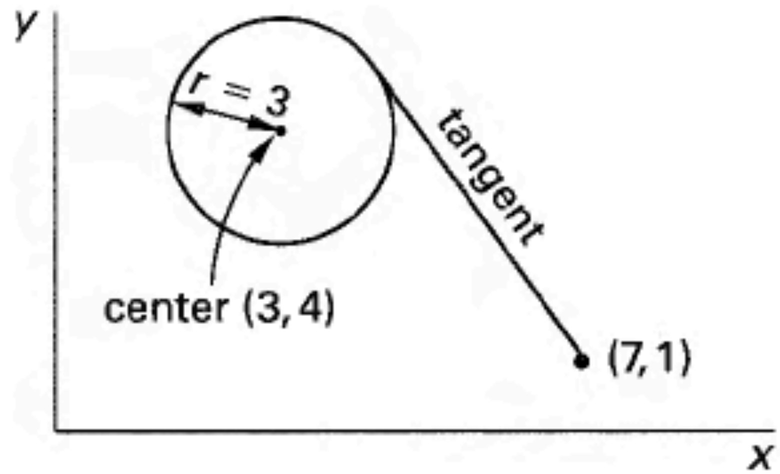
(B) 4

(C) 5

(D) 7

Tangent Line to Circle

What is the length of the line tangent from point $(7, 1)$ to the circle shown?



(A) 3

(B) 4

(C) 5

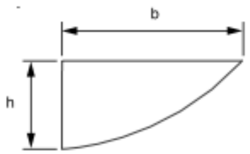
(D) 7

Area (pgs. 20-21)

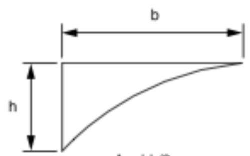
Nomenclature

A = total surface area
P = perimeter
V = volume

Parabola



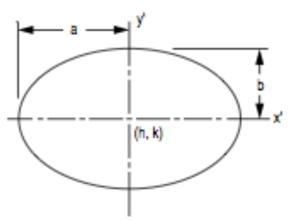
A = 2bh/3



A = bh/3

Ellipse

♦



A = πab

$$P_{approx} = 2\pi\sqrt{(a^2+b^2)/2}$$

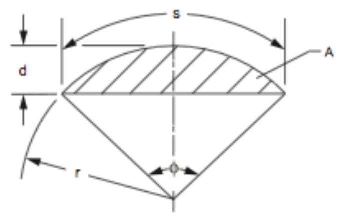
$$P = \pi(a+b)\begin{bmatrix} 1+(1/2)^2\lambda^2+(1/2\times 1/4)^2\lambda^4 \\ +(1/2\times 1/4\times 3/6)^2\lambda^6+(1/2\times 1/4\times 3/6\times 5/8)^2\lambda^8 \\ +(1/2\times 1/4\times 3/6\times 5/8\times 7/10)^2\lambda^{10}+\ldots \end{bmatrix},$$

where

$\lambda = (a-b)/(a+b)$

Circular Segment

♦

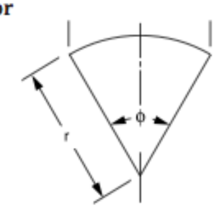


$$A = \left[r^2(\phi - \sin\phi)\right]/2$$

$$\phi = s/r = 2\{\arccos[(r-d)/r]\}$$

Circular Sector

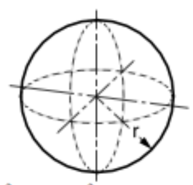
♦



$$A = \phi r^2/2 = sr/2$$
$$\phi = s/r$$

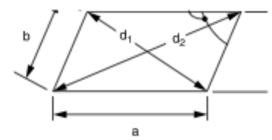
Sphere

♦



$$V = 4\pi r^3/3 = \pi d^3/6$$
$$A = 4\pi r^2 = \pi d^2$$

Parallelogram

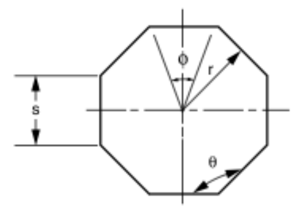


$$P = 2(a+b)$$
$$d_1 = \sqrt{a^2+b^2-2ab(\cos\phi)}$$
$$d_2 = \sqrt{a^2+b^2+2ab(\cos\phi)}$$
$$d_1^2 + d_2^2 = 2(a^2+b^2)$$
$$A = ah = ab(\sin\phi)$$

If $a = b$, the parallelogram is a rhombus.

Regular Polygon (n equal sides)

♦

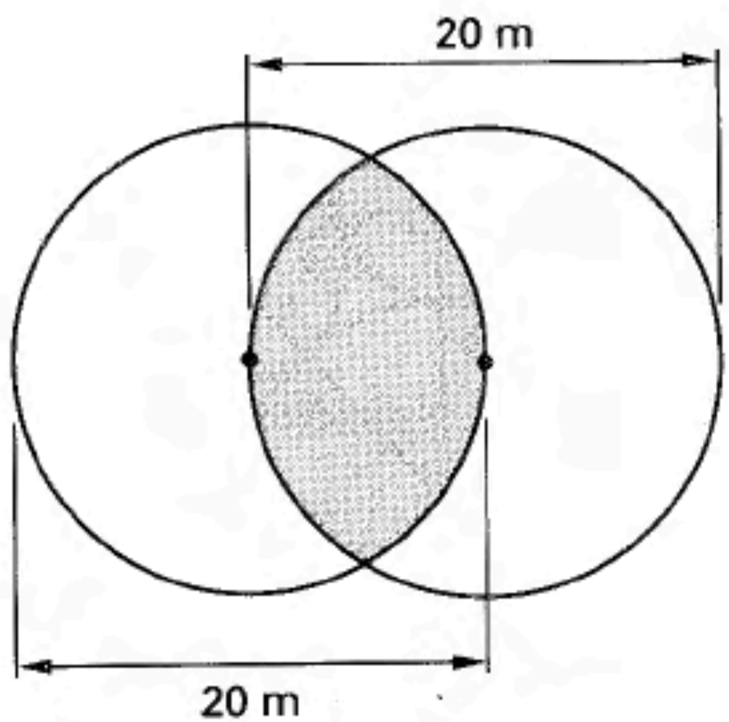


$$\phi = 2\pi/n$$
$$\theta = \left[\frac{\pi(n-2)}{n}\right] = \pi\left(1-\frac{2}{n}\right)$$
$$P = ns$$
$$s = 2r[\tan(\phi/2)]$$
$$A = (nsr)/2$$

→ need to know: circle, rectangle, triangle

Area

Two 20 m diameter circles are placed so that the circumference of each just touches the center of the other.

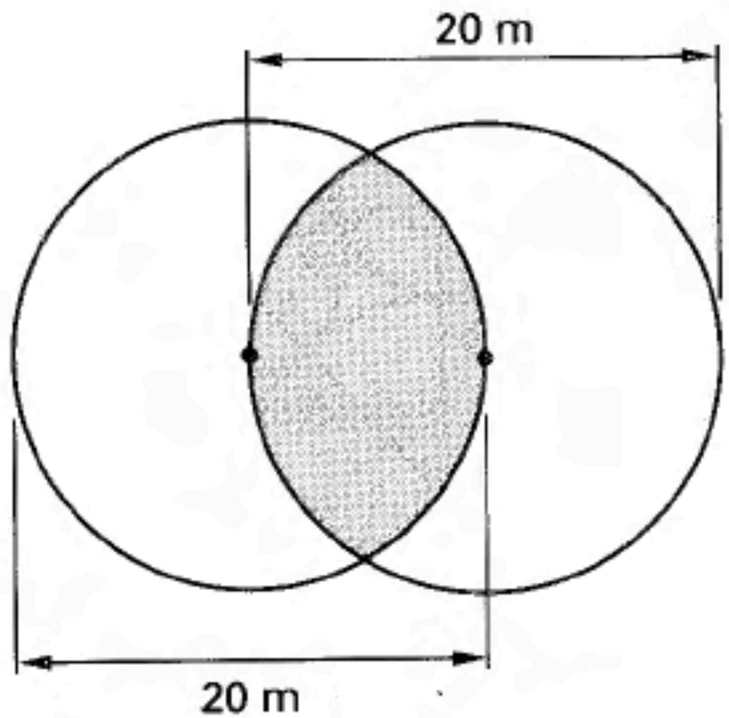


What is most nearly the area of the shared region?

(A) 62 m^2
(B) 110 m^2
(C) 120 m^2
(D) 170 m^2

Area

Two 20 m diameter circles are placed so that the circumference of each just touches the center of the other.



What is most nearly the area of the shared region?

(A) 62 m^2

(B) 110 m^2

(C) 120 m^2

(D) 170 m^2

Area

A regular polygon has six sides, each with a length of 25 cm. What is most nearly the length of the apothem, r?

(A) 10 cm

(B) 15 cm

(C) 20 cm

(D) 22 cm

Area

A regular polygon has six sides, each with a length of 25 cm. What is most nearly the length of the apothem, r?

(A) 10 cm

(B) 15 cm

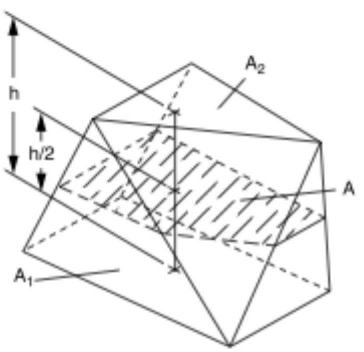
(C) 20 cm

(D) 22 cm

Volume (pgs. 21-22)

Prismoid

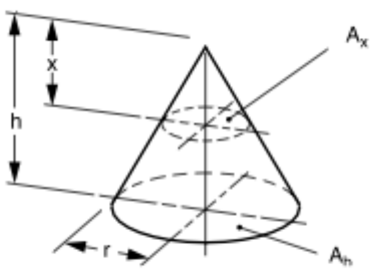
♦



$V = (h/6)(A_1 + A_2 + 4A)$

Right Circular Cone

♦



$V = (\pi r^2 h)/3$

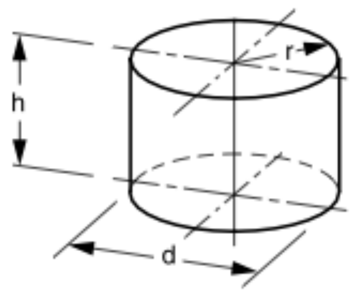
$A =$ side area + base area

$= \pi r\left(r + \sqrt{r^2 + h^2}\right)$

$A_x : A_b = x^2 : h^2$

Right Circular Cylinder

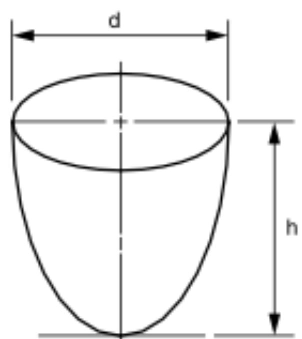
♦



$V = \pi r^2 h = \frac{\pi d^2 h}{4}$

$A =$ side area + end areas $= 2\pi r(h + r)$

Paraboloid of Revolution



$V = \frac{\pi d^2 h}{8}$

Volume

A sphere has a radius of 10 cm. What is approximately the sphere's volume?

(A) 3600 cm^3

(B) 4000 cm^3

(C) 4200 cm^3

(D) 4800 cm^3

Volume

A sphere has a radius of 10 cm. What is approximately the sphere's volume?

(A) 3600 cm^3

(B) 4000 cm^3

(C) 4200 cm^3

(D) 4800 cm^3

Volume

A cone has a height of 100 cm. The cross section of the cone at a distance of 5 cm from the apex is a circle with an area of 20 cm^2. What is most nearly the area of the cone's base?

(A) 5000 cm^2

(B) 6000 cm^2

(C) 8000 cm^2

(D) 9000 cm^2

Volume

A cone has a height of 100 cm. The cross section of the cone at a distance of 5 cm from the apex is a circle with an area of 20 cm^2. What is most nearly the area of the cone's base?

(A) 5000 cm^2

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Algebra

- Logarithms
- Complex Numbers
- Polar Coordinates
- Roots
- Progressions and Series
  - Arithmetic Progression
  - Geometric Progression
  - Properties of Series
  - Power Series

Logarithms (pg. 19)

LOGARITHMS

The logarithm of x to the Base b is defined by

$$\log_b(x) = c, \text{ where } b^c = x$$

Special definitions for $b = e$ or $b = 10$ are:

$$\ln x, \text{ Base} = e$$

$$\log x, \text{ Base} = 10$$

To change from one Base to another:

$$\log_b x = (\log_a x)/(\log_a b)$$

e.g., $\ln x = (\log_{10} x)/(\log_{10} e) = 2.302585\ (\log_{10} x)$

Identities

$$\log_b b^n = n$$

$$\log x^c = c \log x;\ x^c = \text{antilog}\ (c \log x)$$

$$\log xy = \log x + \log y$$

$$\log_b b = 1;\ \log 1 = 0$$

$$\log x/y = \log x - \log y$$

Logarithms

What is the value of $\log_{10} 1000$?

(A) 2

(B) 3

(C) 8

(D) 10

Logarithms

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(B) 3

(C) 8

(D) 10

Logarithms

Which of the following is equal to $(0.001)^{2/3}$?

(A) $\text{antilog}\left(\frac{3}{2}\log 0.001\right)$

(B) $\frac{2}{3}\text{antilog}(\log 0.001)$

(C) $\text{antilog}\left(\log \dfrac{0.001}{\frac{2}{3}}\right)$

(D) $\text{antilog}\left(\frac{2}{3}\log 0.001\right)$

Logarithms

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(D) $\text{antilog}\left(\frac{2}{3}\log 0.001\right)$

Logarithms

Given that $\log_{10} 5 = 0.6990$ and $\log_{10} 9 = 0.9542$, what is the value of $\log_5 9$?

(A) 0.2550

(B) 0.7330

(C) 1.127

(D) 1.365

Logarithms

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Complex Numbers (pg. 19)

Complex numbers may be designated in rectangular form or polar form. In rectangular form, a complex number is written in terms of its real and imaginary components.

$$z = a + jb, \text{ where}$$

a = the real component,

b = the imaginary component, and

$j = \sqrt{-1}$ (some disciplines use $i = \sqrt{-1}$)

In polar form $z = c \angle \theta$ where

$c = \sqrt{a^2 + b^2}$,

$\theta = \tan^{-1}(b/a)$,

$a = c \cos \theta$, and

$b = c \sin \theta$.

Complex numbers can be added and subtracted in rectangular form. If

$$z_1 = a_1 + jb_1 = c_1 (\cos \theta_1 + j\sin \theta_1) = c_1 \angle \theta_1 \text{ and}$$

$$z_2 = a_2 + jb_2 = c_2 (\cos \theta_2 + j\sin \theta_2) = c_2 \angle \theta_2, \text{ then}$$

$$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2) \text{ and}$$

$$z_1 - z_2 = (a_1 - a_2) + j(b_1 - b_2)$$

While complex numbers can be multiplied or divided in rectangular form, it is more convenient to perform these operations in polar form.

$$z_1 \times z_2 = (c_1 \times c_2) \angle (\theta_1 + \theta_2)$$

$$z_1/z_2 = (c_1/c_2) \angle (\theta_1 - \theta_2)$$

The complex conjugate of a complex number $z_1 = (a_1 + jb_1)$ is defined as $z_1{}^* = (a_1 - jb_1)$. The product of a complex number and its complex conjugate is $z_1 z_1{}^* = a_1{}^2 + b_1{}^2$.

Complex Numbers

Which of the following is most nearly equal to $(7+5.2j)/(3+4j)$?

(A) $-0.3+1.8j$

(B) $1.7-0.5j$

(C) $2.3-1.2j$

(D) $2.3+1.3j$

Complex Numbers

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(A) $-0.3 + 1.8j$

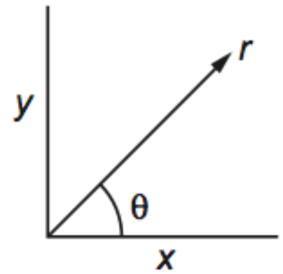
(B) $1.7 - 0.5j$

(C) $2.3 - 1.2j$

(D) $2.3 + 1.3j$

Polar Coordinates (pg. 19)

Polar Coordinate System



$x = r\cos\theta;\ y = r\sin\theta;\ \theta = \arctan(y/x)$

$r = |x + jy| = \sqrt{x^2 + y^2}$

$x + jy = r(\cos\theta + j\sin\theta) = re^{j\theta}$

$$[r_1(\cos\theta_1 + j\sin\theta_1)][r_2(\cos\theta_2 + j\sin\theta_2)] = r_1 r_2[\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2)]$$

$$(x + jy)^n = [r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$$

$$\frac{r_1(\cos\theta_1 + j\sin\theta_1)}{r_2(\cos\theta_2 + j\sin\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)]$$

Euler's Identity

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2},\quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Polar Coordinates

The rectangular coordinates of a complex number are $(4, 6)$. What are the complex number's approximate polar coordinates?

(A) $(4.0, 33°)$

(B) $(4.0, 56°)$

(C) $(7.2, 33°)$

(D) $(7.2, 56°)$

Polar Coordinates

The rectangular coordinates of a complex number are $(4, 6)$. What are the complex number's approximate polar coordinates?

(A) $(4.0, 33°)$

(B) $(4.0, 56°)$

(C) $(7.2, 33°)$

(D) $(7.2, 56°)$

Polar Coordinates

If $j = \sqrt{-1}$, which of the following is equal to j^j?

(A) j^2

(B) e^{2j}

(C) -1

(D) $e^{-\frac{\pi}{2}}$

Polar Coordinates

If $j = \sqrt{-1}$, which of the following is equal to j^j?

(A) j^2

(B) e^{2j}

(C) -1

(D) $e^{-\frac{\pi}{2}}$

Quadratic Equation (pg. 18)

QUADRATIC EQUATION

$$ax^2 + bx + c = 0$$

$$x = \text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Roots: Quadratic Equation

What are the roots of the quadratic equation $-7x + x^2 = -10$?

(A) –5 and 2

(B) –2 and 0.4

(C) 0.4 and 2

(D) 2 and 5

Roots: Quadratic Equation

What are the roots of the quadratic equation $-7x + x^2 = -10$?

(A) –5 and 2

(B) –2 and 0.4

(C) 0.4 and 2

(D) 2 and 5

Progressions and Series (pg. 26)

Arithmetic Progression

To determine whether a given finite sequence of numbers is an arithmetic progression, subtract each number from the following number. If the differences are equal, the series is arithmetic.

1. The first term is a.
2. The common difference is d.
3. The number of terms is n.
4. The last or nth term is l.
5. The sum of n terms is S.

$$l = a + (n - 1)d$$
$$S = n(a + l)/2 = n\,[2a + (n - 1)\,d]/2$$

Geometric Progression

To determine whether a given finite sequence is a geometric progression (G.P.), divide each number after the first by the preceding number. If the quotients are equal, the series is geometric:

1. The first term is a.
2. The common ratio is r.
3. The number of terms is n.
4. The last or nth term is l.
5. The sum of n terms is S.

$$l = ar^{n-1}$$
$$S = a\,(1 - r^n)/(1 - r);\ r \neq 1$$
$$S = (a - rl)/(1 - r);\ r \neq 1$$
$$\lim_{n\to\infty} S_n = a/(1-r);\ r < 1$$

A G.P. converges if $|r| < 1$ and it diverges if $|r| > 1$.

Properties of Series

$$\sum_{i=1}^{n} c = nc; \qquad c = \text{constant}$$
$$\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i$$
$$\sum_{i=1}^{n}\left(x_i + y_i - z_i\right) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} z_i$$
$$\sum_{x=1}^{n} x = \left(n + n^2\right)/2$$

Power Series

$$\Sigma_{i=0}^{\infty}\, a_i\left(x - a\right)^i$$

1. A power series, which is convergent in the interval $-R < x < R$, defines a function of x that is continuous for all values of x within the interval and is said to represent the function in that interval.
2. A power series may be differentiated term by term within its interval of convergence. The resulting series has the same interval of convergence as the original series (except possibly at the end points of the series).
3. A power series may be integrated term by term provided the limits of integration are within the interval of convergence of the series.
4. Two power series may be added, subtracted, or multiplied, and the resulting series in each case is convergent, at least, in the interval common to the two series.
5. Using the process of long division (as for polynomials), two power series may be divided one by the other within their common interval of convergence.

Progressions and Series

What is the sum of the following finite sequence of terms?

$$18, 25, 32, 39, \ldots, 67$$

(A) 181

(B) 213

(C) 234

(D) 340

Progressions and Series

What is the sum of the following finite sequence of terms?

$$18, 25, 32, 39, \ldots, 67$$

(A) 181

(B) 213

(C) 234

(D) 340

Progressions and Series

What is the sum of the following geometric sequence?

$$32, 80, 200, \ldots, 19531.25$$

(A) 21,131.25

(B) 24,718.25

(C) 31,250.00

(D) 32,530.75

Progressions and Series

What is the sum of the following geometric sequence?

$$32, 80, 200, \ldots, 19531.25$$

(A) 21,131.25

(B) 24,718.25

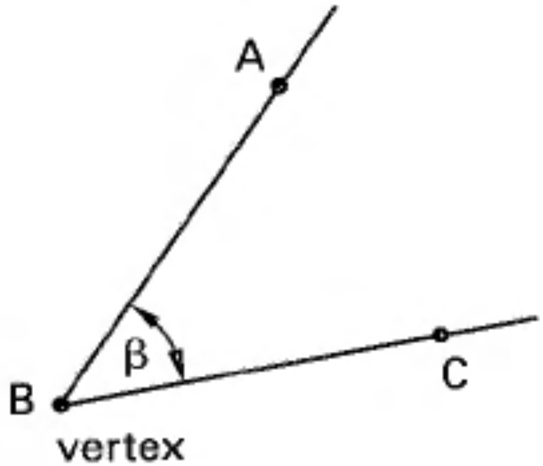
(C) 31,250.00

(D) 32,530.75

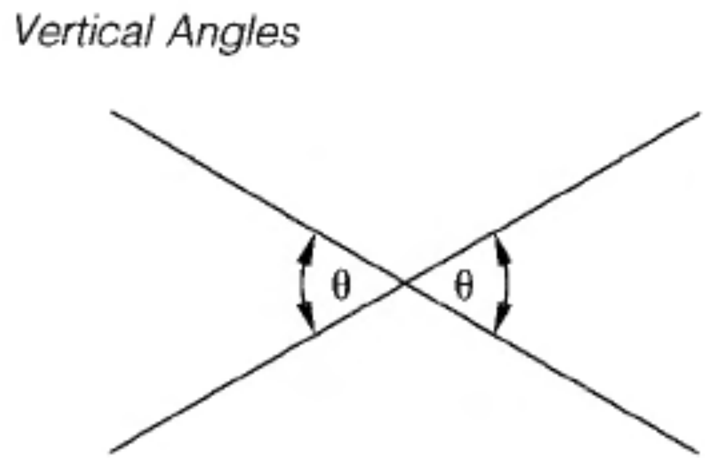
Trigonometry

- Degrees and Radians
- Plane Angles
- Triangles
  - Law of Sines
  - Law of Cosines
- Right Triangles
- General Triangles
- Trigonometric Identities

Angles – Basic Knowledge



- *acute angle:* an angle less than 90° (π/2 rad)
- *obtuse angle:* an angle more than 90° (π/2 rad) but less than 180° (π rad)
- *reflex angle:* an angle more than 180° (π rad) but less than 360° (2π rad)
- *related angle:* an angle that differs from another by some multiple of 90° (π/2 rad)
- *right angle:* an angle equal to 90° (π/2 rad)
- *straight angle:* an angle equal to 180° (π rad); that is, a straight line



radians = degrees * π/180

Triangles (pg. 19)

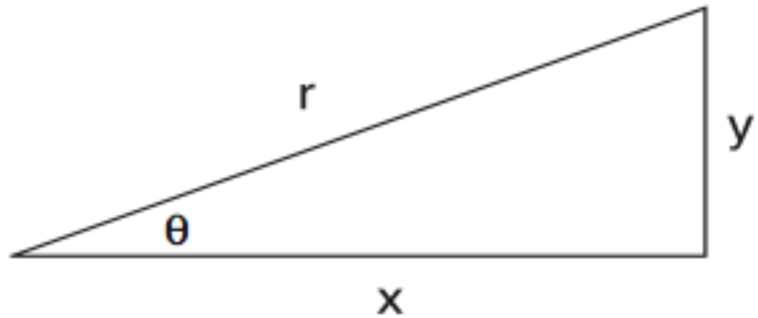
TRIGONOMETRY

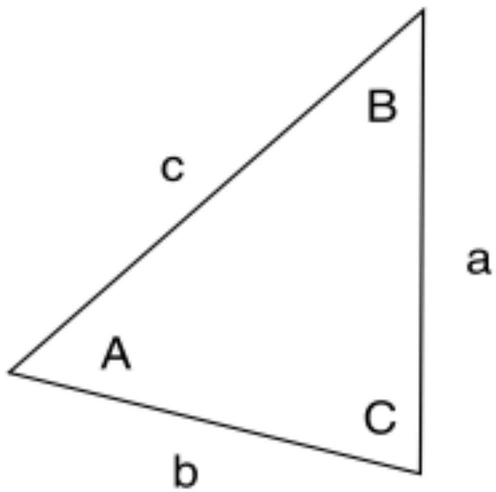
Trigonometric functions are defined using a right triangle.

$\sin\theta = y/r$, $\cos\theta = x/r$

$\tan\theta = y/x$, $\cot\theta = x/y$

$\csc\theta = r/y$, $\sec\theta = r/x$





Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

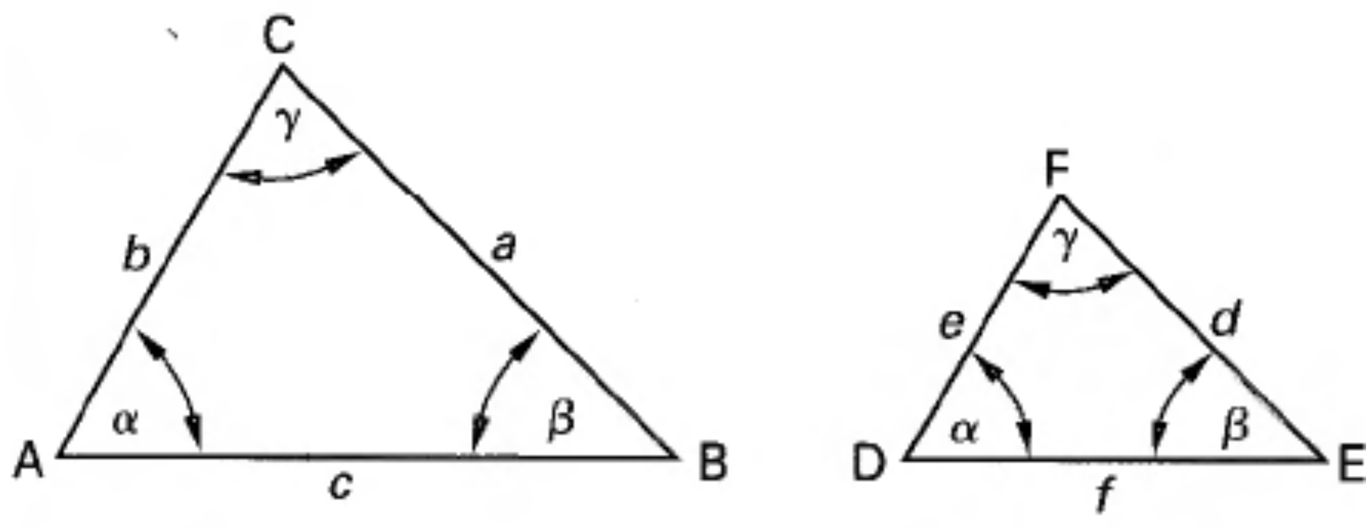
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Triangles – Basic Knowledge

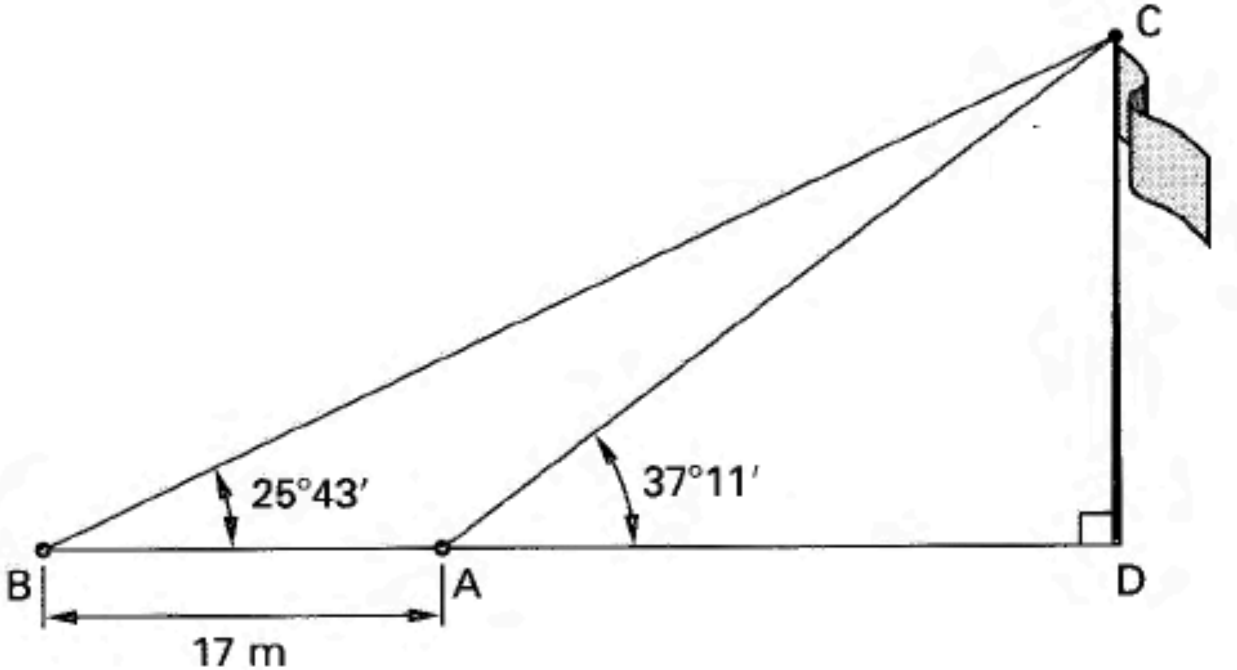
similar triangles



sides are proportional: b/e = c/f = a/d

Triangles

The vertical angle to the top of a flagpole from point A on the ground is observed to be $37^\circ 11'$. The observer walks 17 m directly away from the flagpole from point A to point B and finds the new angle to be $25^\circ 43'$.



(A) 10 m

(B) 22 m

(C) 82 m

(D) 300 m

What is the approximate height of the flagpole?

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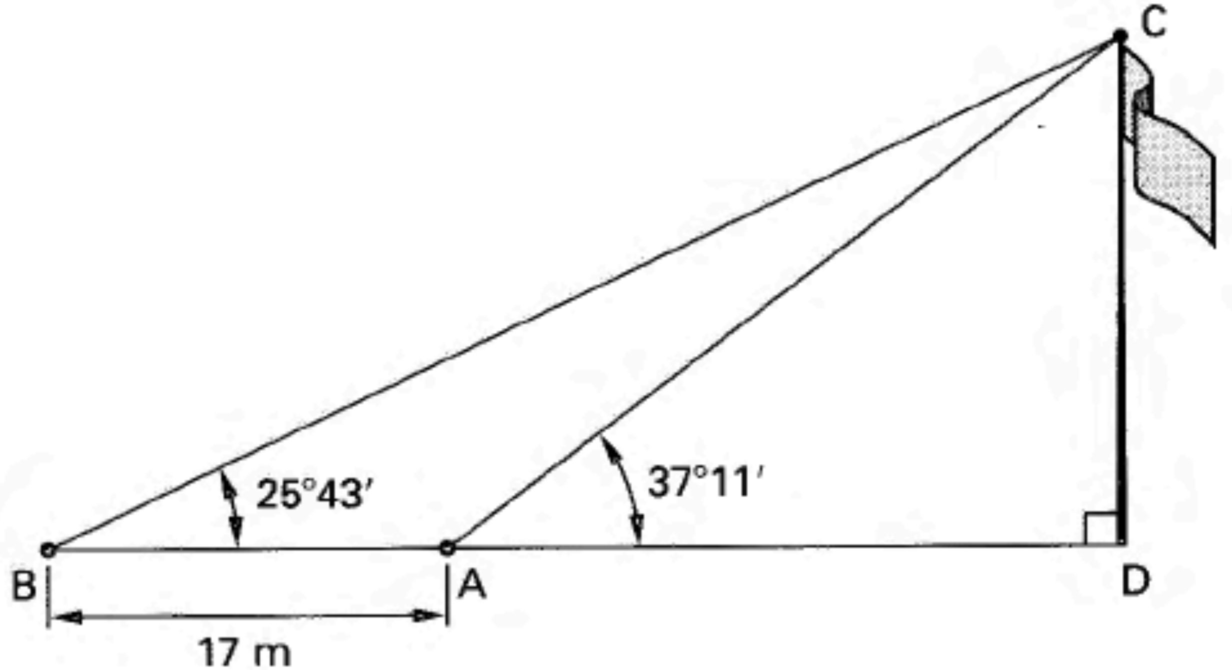
The vertical angle to the top of a flagpole from point A on the ground is observed to be $37^\circ\,11'$. The observer walks 17 m directly away from the flagpole from point A to point B and finds the new angle to be $25^\circ\,43'$.



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What is the approximate height of the flagpole?

Triangles

Three circles of radii 110 m, 140 m, and 220 m are tangent to one another. What are the interior angles of the triangle formed by joining the centers of the circles?

(A) 34.2°, 69.2°, and 76.6°

(B) 36.6°, 69.1°, and 74.3°

(C) 42.2°, 62.5°, and 75.3°

(D) 47.9°, 63.1°, and 69.0°

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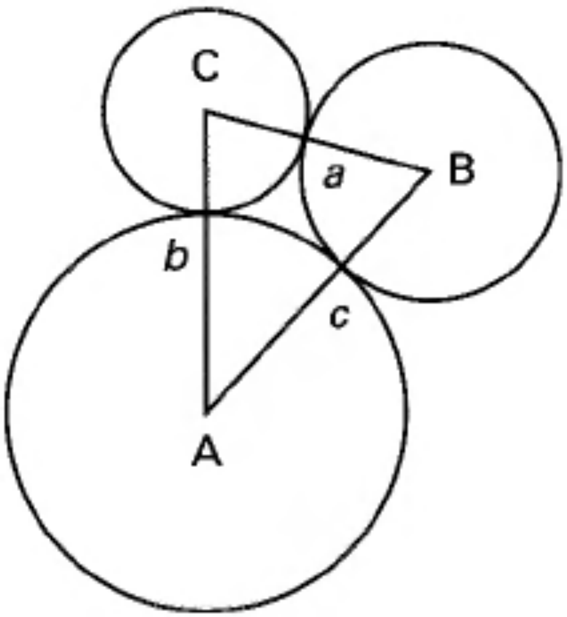

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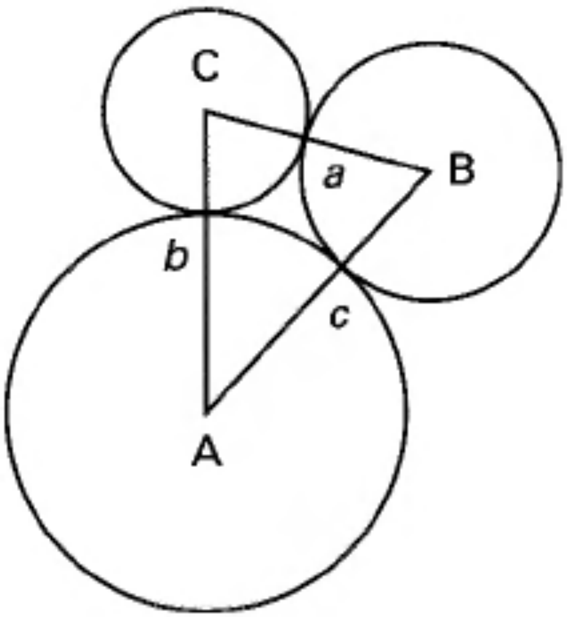

Identities (pg. 20)

Identities

$\cos\theta = \sin(\theta + \pi/2) = -\sin(\theta - \pi/2)$

$\sin\theta = \cos(\theta - \pi/2) = -\cos(\theta + \pi/2)$

$\csc\theta = 1/\sin\theta$

$\sec\theta = 1/\cos\theta$

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = 1/\tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = (2\tan\alpha)/(1 - \tan^2\alpha)$

$\cot 2\alpha = (\cot^2\alpha - 1)/(2\cot\alpha)$

$\tan(\alpha + \beta) = (\tan\alpha + \tan\beta)/(1 - \tan\alpha\tan\beta)$

$\cot(\alpha + \beta) = (\cot\alpha\cot\beta - 1)/(\cot\alpha + \cot\beta)$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = (\tan\alpha - \tan\beta)/(1 + \tan\alpha\tan\beta)$

$\cot(\alpha - \beta) = (\cot\alpha\cot\beta + 1)/(\cot\beta - \cot\alpha)$

$\sin(\alpha/2) = \pm\sqrt{(1 - \cos\alpha)/2}$

$\cos(\alpha/2) = \pm\sqrt{(1 + \cos\alpha)/2}$

$\tan(\alpha/2) = \pm\sqrt{(1 - \cos\alpha)/(1 + \cos\alpha)}$

$\cot(\alpha/2) = \pm\sqrt{(1 + \cos\alpha)/(1 - \cos\alpha)}$

$\sin\alpha\sin\beta = (1/2)[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = (1/2)[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = (1/2)[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\sin\alpha + \sin\beta = 2\sin[(1/2)(\alpha + \beta)]\cos[(1/2)(\alpha - \beta)]$

$\sin\alpha - \sin\beta = 2\cos[(1/2)(\alpha + \beta)]\sin[(1/2)(\alpha - \beta)]$

$\cos\alpha + \cos\beta = 2\cos[(1/2)(\alpha + \beta)]\cos[(1/2)(\alpha - \beta)]$

$\cos\alpha - \cos\beta = -2\sin[(1/2)(\alpha + \beta)]\sin[(1/2)(\alpha - \beta)]$

Identities

Simplify the expression $\cos\theta \sec\theta / \tan\theta$.

(A) 1

(B) $\cot\theta$

(C) $\csc\theta$

(D) $\sin\theta$

Identities

Simplify the expression $\cos\theta\sec\theta/\tan\theta$.

(A) 1

(B) $\cot\theta$

(C) $\csc\theta$

(D) $\sin\theta$

Identities

Which of the following expressions is equivalent to the expression $\csc\theta\cos^3\theta\tan\theta$?

(A) $\sin\theta$

(B) $\cos\theta$

(C) $1-\sin^2\theta$

(D) $1+\sin^2\theta$

Identities

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(D) $1+\sin^2\theta$

Identities

What is an equivalent expression for $\sin 2\alpha$?

(A) $-2\sin\alpha\cos\alpha$

(B) $\frac{1}{2}\sin\alpha\cos\alpha$

(C) $\dfrac{2\sin\alpha}{\sec\alpha}$

(D) $2\sin\alpha\cos\dfrac{\alpha}{2}$

Identities

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(B) $\frac{1}{2}\sin\alpha\cos\alpha$

(C) $\dfrac{2\sin\alpha}{\sec\alpha}$

(D) $2\sin\alpha\cos\dfrac{\alpha}{2}$

Identities

Simplify the following expression.

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos\beta}$$

(A) $\cos\alpha/2$

(B) $2\cos\alpha$

(C) $\sin 2\alpha$

(D) $\sin^2\alpha$

Identities

Simplify the following expression.

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos\beta}$$

(A) $\cos\alpha/2$

(B) $2\cos\alpha$

(C) $\sin 2\alpha$

(D) $\sin^2\alpha$

Calculus

- Differential Calculus
- Critical Points
- Partial Derivatives
- Curvature
- Limits
- Integral Calculus
- Centroids and Moments of Inertia
- Taylor Series

Differential Calculus (pg. 23)

The Derivative

For any function $y = f(x)$,

the derivative $= D_x y = dy/dx = y'$

$$y' = \lim_{\Delta x \to 0} \left[(\Delta y)/(\Delta x)\right]$$

$$= \lim_{\Delta x \to 0} \left\{\left[f(x + \Delta x) - f(x)\right]/(\Delta x)\right\}$$

y' = the slope of the curve $f(x)$.

Derivative and Integral Table (pg. 25)

1. $dc/dx = 0$
2. $dx/dx = 1$
3. $d(cu)/dx = c\,du/dx$
4. $d(u + v - w)/dx = du/dx + dv/dx - dw/dx$
5. $d(uv)/dx = u\,dv/dx + v\,du/dx$
6. $d(uvw)/dx = uv\,dw/dx + uw\,dv/dx + vw\,du/dx$
7. $\dfrac{d(u/v)}{dx} = \dfrac{v\,du/dx - u\,dv/dx}{v^2}$
8. $d(u^n)/dx = nu^{n-1}\,du/dx$
9. $d[f(u)]/dx = \{d[f(u)]/du\}\,du/dx$
10. $du/dx = 1/(dx/du)$
11. $\dfrac{d(\log_a u)}{dx} = (\log_a e)\dfrac{1}{u}\dfrac{du}{dx}$
12. $\dfrac{d(\ln u)}{dx} = \dfrac{1}{u}\dfrac{du}{dx}$
13. $\dfrac{d(a^u)}{dx} = (\ln a)a^u\dfrac{du}{dx}$
14. $d(e^u)/dx = e^u\,du/dx$
15. $d(u^v)/dx = vu^{v-1}\,du/dx + (\ln u)\,u^v\,dv/dx$
16. $d(\sin u)/dx = \cos u\,du/dx$
17. $d(\cos u)/dx = -\sin u\,du/dx$
18. $d(\tan u)/dx = \sec^2 u\,du/dx$
19. $d(\cot u)/dx = -\csc^2 u\,du/dx$
20. $d(\sec u)/dx = \sec u \tan u\,du/dx$
21. $d(\csc u)/dx = -\csc u \cot u\,du/dx$
22. $\dfrac{d(\sin^{-1}u)}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx} \qquad (-\pi/2 \le \sin^{-1}u \le \pi/2)$
23. $\dfrac{d(\cos^{-1}u)}{dx} = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx} \qquad (0 \le \cos^{-1}u \le \pi)$
24. $\dfrac{d(\tan^{-1}u)}{dx} = \dfrac{1}{1+u^2}\dfrac{du}{dx} \qquad (-\pi/2 < \tan^{-1}u < \pi/2)$
25. $\dfrac{d(\cot^{-1}u)}{dx} = -\dfrac{1}{1+u^2}\dfrac{du}{dx} \qquad (0 < \cot^{-1}u < \pi)$
26. $\dfrac{d(\sec^{-1}u)}{dx} = \dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$
    $(0 < \sec^{-1}u < \pi/2)(-\pi \le \sec^{-1}u < -\pi/2)$
27. $\dfrac{d(\csc^{-1}u)}{dx} = -\dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}$
    $(0 < \csc^{-1}u \le \pi/2)(-\pi < \csc^{-1}u \le -\pi/2)$

1. $\int d\,f(x) = f(x)$
2. $\int dx = x$
3. $\int a f(x)\,dx = a \int f(x)\,dx$
4. $\int [u(x) \pm v(x)]\,dx = \int u(x)\,dx \pm \int v(x)\,dx$
5. $\int x^m dx = \dfrac{x^{m+1}}{m+1} \qquad (m \ne -1)$
6. $\int u(x)\,dv(x) = u(x)\,v(x) - \int v(x)\,du(x)$
7. $\int \dfrac{dx}{ax+b} = \dfrac{1}{a}\ln|ax+b|$
8. $\int \dfrac{dx}{\sqrt{x}} = 2\sqrt{x}$
9. $\int a^x dx = \dfrac{a^x}{\ln a}$
10. $\int \sin x\,dx = -\cos x$
11. $\int \cos x\,dx = \sin x$
12. $\int \sin^2 x dx = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$
13. $\int \cos^2 x dx = \dfrac{x}{2} + \dfrac{\sin 2x}{4}$
14. $\int x \sin x\,dx = \sin x - x \cos x$
15. $\int x \cos x\,dx = \cos x + x \sin x$
16. $\int \sin x \cos x\,dx = (\sin^2 x)/2$
17. $\int \sin ax \cos bx\,dx = -\dfrac{\cos(a-b)x}{2(a-b)} - \dfrac{\cos(a+b)x}{2(a+b)} \quad (a^2 \ne b^2)$
18. $\int \tan x\,dx = -\ln|\cos x| = \ln|\sec x|$
19. $\int \cot x\,dx = -\ln|\csc x| = \ln|\sin x|$
20. $\int \tan^2 x\,dx = \tan x - x$
21. $\int \cot^2 x\,dx = -\cot x - x$
22. $\int e^{ax}\,dx = (1/a)\,e^{ax}$
23. $\int xe^{ax}\,dx = (e^{ax}/a^2)(ax - 1)$
24. $\int \ln x\,dx = x\,[\ln(x) - 1] \qquad (x > 0)$
25. $\int \dfrac{dx}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} \qquad (a \ne 0)$
26. $\int \dfrac{dx}{ax^2+c} = \dfrac{1}{\sqrt{ac}}\tan^{-1}\left(x\sqrt{\dfrac{a}{c}}\right), \qquad (a > 0, c > 0)$

27a. $\int \dfrac{dx}{ax^2+bx+c} = \dfrac{2}{\sqrt{4ac-b^2}}\tan^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}}$
    $(4ac - b^2 > 0)$

27b. $\int \dfrac{dx}{ax^2+bx+c} = \dfrac{1}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right|$
    $(b^2 - 4ac > 0)$

27c. $\int \dfrac{dx}{ax^2+bx+c} = -\dfrac{2}{2ax+b}, \qquad (b^2 - 4ac = 0)$

- Derivatives of polynomials missing
- Product rule of differentiation
- Integration by parts

Differential Calculus

What is the slope of the curve $y = 10x^2 - 3x - 1$ when it crosses the positive part of the x-axis?

(A) 3/20

(B) 1/5

(C) 1/3

(D) 7

Differential Calculus

What is the slope of the curve $y = 10x^2 - 3x - 1$ when it crosses the positive part of the x-axis?

(A) 3/20

(B) 1/5

(C) 1/3

(D) 7

Differential Calculus

Evaluate dy/dx for the following expression.

$$y = e^{-x} \sin 2x$$

(A) $e^{-x}(2\cos 2x - \sin 2x)$

(B) $-e^{-x}(2\sin 2x + \cos 2x)$

(C) $e^{-x}(2\sin 2x + \cos 2x)$

(D) $-e^{-x}(2\cos 2x - \sin 2x)$

Differential Calculus

Evaluate dy/dx for the following expression.

$$y = e^{-x} \sin 2x$$

(A) $e^{-x}(2\cos 2x - \sin 2x)$

(B) $-e^{-x}(2\sin 2x + \cos 2x)$

(C) $e^{-x}(2\sin 2x + \cos 2x)$

(D) $-e^{-x}(2\cos 2x - \sin 2x)$

Critical Points (pg. 23)

<u>Test for a Maximum</u>

$y = f(x)$ is a maximum for
$x = a$, if $f'(a) = 0$ and $f''(a) < 0$.

<u>Test for a Minimum</u>

$y = f(x)$ is a minimum for
$x = a$, if $f'(a) = 0$ and $f''(a) > 0$.

<u>Test for a Point of Inflection</u>

$y = f(x)$ has a point of inflection at $x = a$,
if $f''(a) = 0$, and
if $f''(x)$ changes sign as x increases through
$x = a$.

Critical Points

What is the maximum value of the function $f(x) = -x^2 - 8x + 1$?

(A) 1

(B) 4

(C) 8

(D) 17

Critical Points

What is the maximum value of the function $f(x) = -x^2 - 8x + 1$?

(A) 1

(B) 4

(C) 8

(D) 17

Critical Points

What is the minimum value of the function $f(x) = 3x^2 + 3x - 5$?

(A) -12.0

(B) -8.0

(C) -5.75

(D) -5.00

Critical Points

What is the minimum value of the function $f(x) = 3x^2 + 3x - 5$?

(A) -12.0

(B) -8.0

(C) -5.75

(D) -5.00

Partial Derivatives (pg. 23)

The Partial Derivative

In a function of two independent variables x and y, a derivative with respect to one of the variables may be found if the other variable is *assumed* to remain constant. If *y is kept fixed*, the function

$$z = f(x, y)$$

becomes a function of the *single variable* x, and its derivative (if it exists) can be found. This derivative is called the *partial derivative of z with respect to x*. The partial derivative with respect to x is denoted as follows:

$$\frac{\partial z}{\partial x} = \frac{\partial f(x,y)}{\partial x}$$

Partial Derivatives

What is the partial derivative with respect to x of the following function?

$$z = e^{xy}$$

(A) e^{xy}

(B) $\dfrac{e^{xy}}{x}$

(C) $\dfrac{e^{xy}}{y}$

(D) ye^{xy}

Partial Derivatives

What is the partial derivative with respect to x of the following function?

$$z = e^{xy}$$

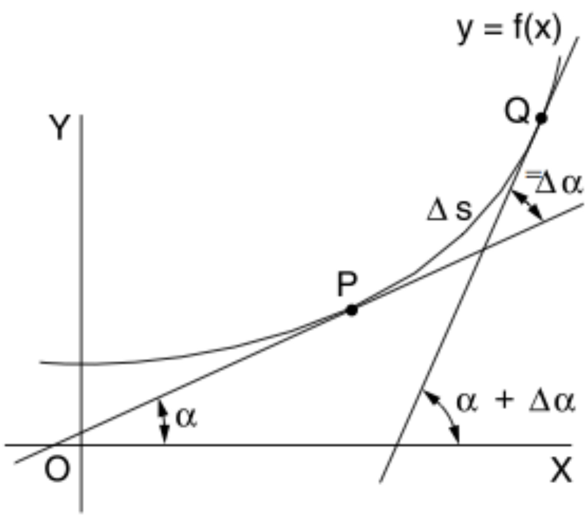
(A) e^{xy}

(B) $\dfrac{e^{xy}}{x}$

(C) $\dfrac{e^{xy}}{y}$

(D) ye^{xy}

Curvature (pg. 24)



The curvature K of a curve at P is the limit of its average curvature for the arc PQ as Q approaches P. This is also expressed as: the curvature of a curve at a given point is the rate-of-change of its inclination with respect to its arc length.

$$K = \lim_{\Delta s \to 0} \frac{\Delta\alpha}{\Delta s} = \frac{d\alpha}{ds}$$

Curvature in Rectangular Coordinates

$$K = \frac{y''}{\left[1 + (y')^2\right]^{3/2}}$$

When it may be easier to differentiate the function with respect to y rather than x, the notation x' will be used for the derivative.

$$x' = dx/dy$$

$$K = \frac{-x''}{\left[1 + (x')^2\right]^{3/2}}$$

The Radius of Curvature

The *radius of curvature* R at any point on a curve is defined as the absolute value of the reciprocal of the curvature K at that point.

$$R = \frac{1}{|K|} \qquad (K \neq 0)$$

$$R = \left| \frac{\left[1 + (y')^2\right]^{3/2}}{|y''|} \right| \qquad (y'' \neq 0)$$

Curvature

What is the approximate radius of curvature of the function $f(x)$ at the point $(x, y) = (8, 16)$?

$$f(x) = x^2 + 6x - 96$$

(A) 1.9×10^{-4}

(B) 9.8

(C) 96

(D) 5300

Curvature

What is the approximate radius of curvature of the function $f(x)$ at the point $(x, y) = (8, 16)$?

$$f(x) = x^2 + 6x - 96$$

(A) 1.9×10^{-4}

(B) 9.8

(C) 96

(D) 5300

Limits (pg. 24)

L'Hospital's Rule (L'Hôpital's Rule)

If the fractional function $f(x)/g(x)$ assumes one of the indeterminate forms 0/0 or ∞/∞ (where α is finite or infinite), then

$$\underset{x \to \alpha}{\text{limit}}\ f(x)/g(x)$$

is equal to the first of the expressions

$$\underset{x \to \alpha}{\text{limit}}\ \frac{f'(x)}{g'(x)},\ \underset{x \to \alpha}{\text{limit}}\ \frac{f''(x)}{g''(x)},\ \underset{x \to \alpha}{\text{limit}}\ \frac{f'''(x)}{g'''(x)}$$

which is not indeterminate, provided such first indicated limit exists.

Limits

Evaluate the following limit.

$$\lim_{x \to 0} \frac{1 - e^{3x}}{4x}$$

(A) $-\infty$

(B) $-3/4$

(C) 0

(D) $1/4$

Limits

Evaluate the following limit.

$$\operatorname*{limit}_{x\to 0} \frac{1-e^{3x}}{4x}$$

(A) $-\infty$

(B) $-3/4$

(C) 0

(D) $1/4$

Integral Calculus (pg. 24)

INTEGRAL CALCULUS

The definite integral is defined as:

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x_i = \int_a^b f(x)\, dx$$

Also, $\Delta x_i \to 0$ for all i.

A table of derivatives and integrals is available in the Derivatives and Indefinite Integrals section. The integral equations can be used along with the following methods of integration:

A. Integration by Parts (integral equation #6),

B. Integration by Substitution, and

C. Separation of Rational Fractions into Partial Fractions.

Integral Calculus

What is the approximate total area bounded by $y = \sin x$ over the interval $0 \leq x \leq 2\pi$? (x is in radians.)

(A) 0

(B) $\pi/2$

(C) 2

(D) 4

Integral Calculus

What is the approximate total area bounded by $y = \sin x$ over the interval $0 \leq x \leq 2\pi$? (x is in radians.)

(A) 0

(B) $\pi/2$

(C) 2

(D) 4

Derivative and Integral Table (pg. 25)

1. $dc/dx = 0$
2. $dx/dx = 1$
3. $d(cu)/dx = c\, du/dx$
4. $d(u + v - w)/dx = du/dx + dv/dx - dw/dx$
5. $d(uv)/dx = u\, dv/dx + v\, du/dx$
6. $d(uvw)/dx = uv\, dw/dx + uw\, dv/dx + vw\, du/dx$
7. $\frac{d(u/v)}{dx} = \frac{v\, du/dx - u\, dv/dx}{v^2}$
8. $d(u^n)/dx = nu^{n-1}\, du/dx$
9. $d[f(u)]/dx = \{d[f(u)]/du\}\, du/dx$
10. $du/dx = 1/(dx/du)$
11. $\frac{d(\log_a u)}{dx} = (\log_a e)\frac{1}{u}\frac{du}{dx}$
12. $\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}$
13. $\frac{d(a^u)}{dx} = (\ln a)a^u\frac{du}{dx}$
14. $d(e^u)/dx = e^u\, du/dx$
15. $d(u^v)/dx = vu^{v-1}\, du/dx + (\ln u)\, u^v\, dv/dx$
16. $d(\sin u)/dx = \cos u\, du/dx$
17. $d(\cos u)/dx = -\sin u\, du/dx$
18. $d(\tan u)/dx = \sec^2 u\, du/dx$
19. $d(\cot u)/dx = -\csc^2 u\, du/dx$
20. $d(\sec u)/dx = \sec u \tan u\, du/dx$
21. $d(\csc u)/dx = -\csc u \cot u\, du/dx$
22. $\frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\quad(-\pi/2 \le \sin^{-1}u \le \pi/2)$
23. $\frac{d(\cos^{-1}u)}{dx} = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\quad(0 \le \cos^{-1}u \le \pi)$
24. $\frac{d(\tan^{-1}u)}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$ $\quad(-\pi/2 < \tan^{-1}u < \pi/2)$
25. $\frac{d(\cot^{-1}u)}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$ $\quad(0 < \cot^{-1}u < \pi)$
26. $\frac{d(\sec^{-1}u)}{dx} = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$
    $(0 < \sec^{-1}u < \pi/2)(-\pi \le \sec^{-1}u < -\pi/2)$
27. $\frac{d(\csc^{-1}u)}{dx} = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$
    $(0 < \csc^{-1}u \le \pi/2)(-\pi < \csc^{-1}u \le -\pi/2)$

1. $\int d\,f(x) = f(x)$
2. $\int dx = x$
3. $\int a\,f(x)\,dx = a\int f(x)\,dx$
4. $\int [u(x) \pm v(x)]\,dx = \int u(x)\,dx \pm \int v(x)\,dx$
5. $\int x^m dx = \frac{x^{m+1}}{m+1}$ $\quad(m \ne -1)$
6. $\int u(x)\,dv(x) = u(x)\,v(x) - \int v(x)\,du(x)$
7. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$
8. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$
9. $\int a^x dx = \frac{a^x}{\ln a}$
10. $\int \sin x\,dx = -\cos x$
11. $\int \cos x\,dx = \sin x$
12. $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$
13. $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}$
14. $\int x \sin x\,dx = \sin x - x\cos x$
15. $\int x \cos x\,dx = \cos x + x \sin x$
16. $\int \sin x \cos x\,dx = (\sin^2 x)/2$
17. $\int \sin ax \cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$ $(a^2 \ne b^2)$
18. $\int \tan x\,dx = -\ln|\cos x| = \ln|\sec x|$
19. $\int \cot x\,dx = -\ln|\csc x| = \ln|\sin x|$
20. $\int \tan^2 x\,dx = \tan x - x$
21. $\int \cot^2 x\,dx = -\cot x - x$
22. $\int e^{ax}\,dx = (1/a)\,e^{ax}$
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24. $\int \ln x\,dx = x\,[\ln(x) - 1]$ $\quad(x > 0)$
25. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$ $\quad(a \ne 0)$
26. $\int \frac{dx}{ax^2+c} = \frac{1}{\sqrt{ac}}\tan^{-1}\left(x\sqrt{\frac{a}{c}}\right),$ $\quad(a > 0, c > 0)$

27a. $\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$
$(4ac - b^2 > 0)$

27b. $\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right|$
$(b^2 - 4ac > 0)$

27c. $\int \frac{dx}{ax^2+bx+c} = -\frac{2}{2ax+b},$ $\quad(b^2 - 4ac = 0)$

Centroids and Moments of Inertia (pg. 26)

The *location of the centroid of an area*, bounded by the axes and the function $y = f(x)$, can be found by integration.

$$x_c = \frac{\int x dA}{A}$$

$$y_c = \frac{\int y dA}{A}$$

$$A = \int f(x) dx$$

$$dA = f(x) dx = g(y) dy$$

The *first moment of area* with respect to the y-axis and the x-axis, respectively, are:

$$M_y = \int x\, dA = x_c A$$

$$M_x = \int y\, dA = y_c A$$

The *moment of inertia* (*second moment of area*) with respect to the y-axis and the x-axis, respectively, are:

$$I_y = \int x^2\, dA$$

$$I_x = \int y^2\, dA$$

The moment of inertia taken with respect to an axis passing through the area's centroid is the *centroidal moment of inertia*. The *parallel axis theorem* for the moment of inertia with respect to another axis parallel with and located d units from the centroidal axis is expressed by

$$I_{\text{parallel axis}} = I_c + Ad^2$$

In a plane, $J = \int r^2 dA = I_x + I_y$

Centroids and Moments of Inertia

What is most nearly the x-coordinate of the centroid of the area bounded by $y=0$, $f(x)$, $x=0$, and $x=20$?

$$f(x) = x^3 + 7x^2 - 5x + 6$$

(A) 7.6

(B) 9.4

(C) 14

(D) 16

Centroids and Moments of Inertia

What is most nearly the x-coordinate of the centroid of the area bounded by $y = 0$, $f(x)$, $x = 0$, and $x = 20$?

$$f(x) = x^3 + 7x^2 - 5x + 6$$

(A) 7.6

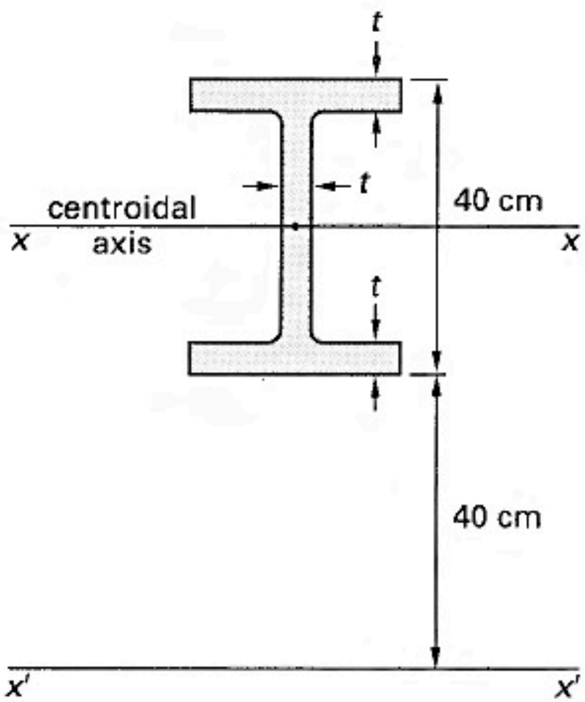
(B) 9.4

(C) 14

(D) 16

Centroids and Moments of Inertia

The moment of inertia about the x'-axis of the cross section shown is 334 000 cm^4. The cross-sectional area is 86 cm^2, and the thicknesses of the web and the flanges are the same.



What is most nearly the moment of inertia about the centroidal axis?

(A) 2.4×10^4 cm^4

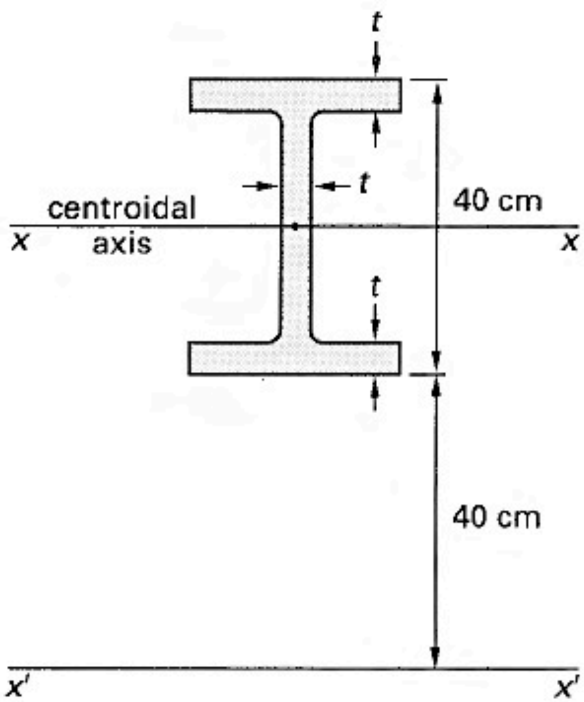
(B) 7.4×10^4 cm^4

(C) 2.0×10^5 cm^4

(D) 6.4×10^5 cm^4

Centroids and Moments of Inertia

The moment of inertia about the x'-axis of the cross section shown is 334 000 cm^4. The cross-sectional area is 86 cm^2, and the thicknesses of the web and the flanges are the same.



What is most nearly the moment of inertia about the centroidal axis?

(A) 2.4×10^4 cm^4

(B) 7.4×10^4 cm^4

(C) 2.0×10^5 cm^4

(D) 6.4×10^5 cm^4

Taylor Series (pg. 26)

Taylor's Series

$$f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x - a)^n + \ldots$$

is called *Taylor's series*, and the function $f(x)$ is said to be expanded about the point a in a Taylor's series.

If $a = 0$, the Taylor's series equation becomes a *Maclaurin's series*.

Taylor Series

Taylor's series is used to expand the function $f(x)$ about $a = 0$ to obtain $f(b)$.

$$f(x) = \frac{1}{3x^3 + 4x + 8}$$

What are the first two terms of Taylor's series?

(A) $\frac{1}{16} + \frac{b}{8}$

(B) $\frac{1}{8} - \frac{b}{16}$

(C) $\frac{1}{8} + \frac{b}{16}$

(D) $\frac{1}{4} - \frac{b}{16}$

Taylor Series

Taylor's series is used to expand the function $f(x)$ about $a = 0$ to obtain $f(b)$.

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(C) $\frac{1}{8} + \frac{b}{16}$

(D) $\frac{1}{4} - \frac{b}{16}$

Differential Equations

- Ordinary Linear Differential Equations
- 1st Order Homogenous ODEs
- 2nd Order Homogenous ODEs
- 1st Order Nonhomogeneous ODEs
- Fourier Transform
- Fourier Series
- Laplace Transform

Ordinary Linear Differential Eqn (pg. 27)

A common class of ordinary linear differential equations is

$$b_n \frac{d^n y(x)}{dx^n} + \ldots + b_1 \frac{dy(x)}{dx} + b_0 y(x) = f(x)$$

where $b_n, \ldots, b_i, \ldots, b_1, b_0$ are constants.

When the equation is a homogeneous differential equation, $f(x) = 0$, the solution is

$$y_h(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x} + \ldots + C_i e^{r_i x} + \ldots + C_n e^{r_n x}$$

where r_n is the nth distinct root of the characteristic polynomial $P(x)$ with

$$P(r) = b_n r^n + b_{n-1} r^{n-1} + \ldots + b_1 r + b_0$$

If the root $r_1 = r_2$, then $C_2 e^{r_2 x}$ is replaced with $C_2 x e^{r_1 x}$.

Higher orders of multiplicity imply higher powers of x. The complete solution for the differential equation is

$$y(x) = y_h(x) + y_p(x),$$

where $y_p(x)$ is any particular solution with $f(x)$ present. If $f(x)$ has $e^{r_n x}$ terms, then resonance is manifested. Furthermore, specific $f(x)$ forms result in specific $y_p(x)$ forms, some of which are:

$f(x)$	$y_p(x)$
A	B
$Ae^{\alpha x}$	$Be^{\alpha x}$, $\alpha \neq r_n$
$A_1 \sin \omega x + A_2 \cos \omega x$	$B_1 \sin \omega x + B_2 \cos \omega x$

If the independent variable is time t, then transient dynamic solutions are implied.

Ordinary Linear Differential Eqn

Which of the following is NOT a linear differential equation?

(A) $5\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = 4te^{-7t}$

(B) $5\frac{d^2y}{dt^2} - 8t^2\frac{dy}{dt} + 16y = 0$

(C) $5\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = \frac{dy}{dy}$

(D) $5\left(\frac{dy}{dt}\right)^2 - 8\frac{dy}{dt} + 16y = 0$

Ordinary Linear Differential Eqn

Which of the following is NOT a linear differential equation?

(A) $5\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = 4te^{-7t}$

(B) $5\frac{d^2y}{dt^2} - 8t^2\frac{dy}{dt} + 16y = 0$

(C) $5\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = \frac{dy}{dy}$

(D) $5\left(\frac{dy}{dt}\right)^2 - 8\frac{dy}{dt} + 16y = 0$

Ordinary Linear Differential Eqn

Which of the following is NOT a linear differential equation?

(A) $5\dfrac{d^2y}{dt^2} - 8\dfrac{dy}{dt} + 16y = 4te^{-7t}$

(B) $5\dfrac{d^2y}{dt^2} - 8t^2\dfrac{dy}{dt} + 16y = 0$

(C) $5\dfrac{d^2y}{dt^2} - 8\dfrac{dy}{dt} + 16y = \dfrac{dy}{dy}$

(D) $5\left(\dfrac{dy}{dt}\right)^2 - 8\dfrac{dy}{dt} + 16y = 0$

1^{st} Order Homogeneous ODE (pg. 27)

First-Order Linear Homogeneous Differential Equations with Constant Coefficients

$y' + ay = 0$, where a is a real constant:

Solution, $y = Ce^{-at}$

where C = a constant that satisfies the initial conditions.

1st Order Homogeneous ODE

Which of the following is the general solution to the differential equation and boundary conditions?

$$\frac{dy}{dt} - 5y = 0$$
$$y(0) = 3$$

(A) $-\frac{1}{3}e^{-5t}$

(B) $3e^{5t}$

(C) $5e^{-3t}$

(D) $\frac{1}{5}e^{-3t}$

1st Order Homogeneous ODE

Which of the following is the general solution to the differential equation and boundary conditions?

$$\frac{dy}{dt} - 5y = 0$$
$$y(0) = 3$$

(A) $-\frac{1}{3}e^{-5t}$

(B) $3e^{5t}$

(C) $5e^{-3t}$

(D) $\frac{1}{5}e^{-3t}$

2nd Order Homogeneous ODE (pg. 27)

Second-Order Linear Homogeneous Differential Equations with Constant Coefficients

An equation of the form

$$y'' + ay' + by = 0$$

can be solved by the method of undetermined coefficients where a solution of the form $y = Ce^{rx}$ is sought. Substitution of this solution gives

$$(r^2 + ar + b)\,Ce^{rx} = 0$$

and since Ce^{rx} cannot be zero, the characteristic equation must vanish or

$$r^2 + ar + b = 0$$

The roots of the characteristic equation are

$$r_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

and can be real and distinct for $a^2 > 4b$, real and equal for $a^2 = 4b$, and complex for $a^2 < 4b$.

If $a^2 > 4b$, the solution is of the form (overdamped)

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

If $a^2 = 4b$, the solution is of the form (critically damped)

$$y = (C_1 + C_2 x)e^{r_1 x}$$

If $a^2 < 4b$, the solution is of the form (underdamped)

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x), \text{ where}$$

$$\alpha = -a/2$$

$$\beta = \frac{\sqrt{4b - a^2}}{2}$$

2nd Order Homogeneous ODE

What is the general solution to the following homogeneous differential equation?

$$y'' - 8y' + 16y = 0$$

(A) $y = C_1 e^{4x}$

(B) $y = (C_1 + C_2 x)e^{4x}$

(C) $y = C_1 e^{-4x} + C_2 e^{4x}$

(D) $y = C_1 e^{2x} + C_2 e^{4x}$

2nd Order Homogeneous ODE

What is the general solution to the following homogeneous differential equation?

$$y'' - 8y' + 16y = 0$$

(A) $y = C_1 e^{4x}$

(B) $y = (C_1 + C_2 x)e^{4x}$

(C) $y = C_1 e^{-4x} + C_2 e^{4x}$

(D) $y = C_1 e^{2x} + C_2 e^{4x}$

1st Order Nonhomogeneous ODE (pg. 27)

First-Order Linear Nonhomogeneous Differential Equations

$$\tau \frac{dy}{dt} + y = Kx(t) \qquad x(t) = \begin{cases} A & t < 0 \\ B & t > 0 \end{cases}$$

$$y(0) = KA$$

τ is the time constant
K is the gain
The solution is

$$y(t) = KA + (KB - KA)\left(1 - \exp\left(\frac{-t}{\tau}\right)\right) \text{ or}$$

$$\frac{t}{\tau} = \ln\left[\frac{KB - KA}{KB - y}\right]$$

1st Order Nonhomogeneous ODE

A spring-mass-dashpot system starting from a motionless state is acted upon by a step function. The response is described by the differential equation in which time, t, is given in seconds measured from the application of the ramp function.

$$\frac{dy}{dt} + 2y = 2u(0) \quad [y(0) = 0]$$

How long will it take for the system to reach 63% of its final value?

(A) 0.25 s

(B) 0.50 s

(C) 1.0 s

(D) 2.0 s

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Fourier Series (pg. 28)

Every periodic function $f(t)$ which has the period $T = 2\pi/\omega_0$ and has certain continuity conditions can be represented by a series plus a constant

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right]$$

The above holds if $f(t)$ has a continuous derivative $f'(t)$ for all t. It should be noted that the various sinusoids present in the series are orthogonal on the interval 0 to T and as a result the coefficients are given by

$$a_0 = (1/T)\int_0^T f(t)\,dt$$
$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_0 t)\,dt \qquad n = 1, 2, \ldots$$
$$b_n = (2/T)\int_0^T f(t)\sin(n\omega_0 t)\,dt \qquad n = 1, 2, \ldots$$

The constants a_n and b_n are the *Fourier coefficients* of $f(t)$ for the interval 0 to T and the corresponding series is called the *Fourier series of* $f(t)$ over the same interval.

The integrals have the same value when evaluated over any interval of length T.

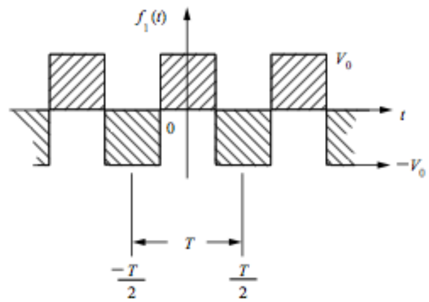
If a Fourier series representing a periodic function is truncated after term $n = N$ the mean square value F_N^2 of the truncated series is given by Parseval's relation. This relation says that the mean-square value is the sum of the mean-square values of the Fourier components, or

$$F_N^2 = a_0^2 + (1/2)\sum_{n=1}^{N}\left(a_n^2 + b_n^2\right)$$

and the RMS value is then defined to be the square root of this quantity or F_N.

Three useful and common Fourier series forms are defined in terms of the following graphs (with $\omega_0 = 2\pi/T$).

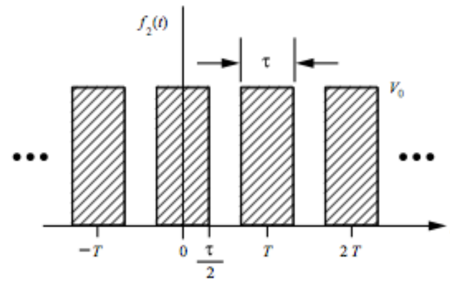
Given:



then

$$f_1(t) = \sum_{\substack{n=1 \\ (n\ \text{odd})}}^{\infty}(-1)^{(n-1)/2}\left(4V_0/n\pi\right)\cos(n\omega_0 t)$$

Given:

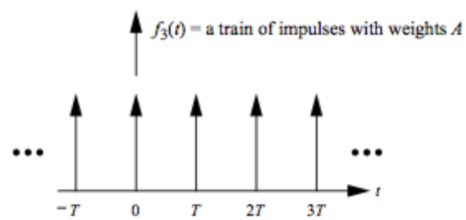


then

$$f_2(t) = \frac{V_0\tau}{T} + \frac{2V_0\tau}{T}\sum_{n=1}^{\infty}\frac{\sin(n\pi\tau/T)}{(n\pi\tau/T)}\cos(n\omega_0 t)$$

$$f_2(t) = \frac{V_0\tau}{T}\sum_{n=-\infty}^{\infty}\frac{\sin(n\pi\tau/T)}{(n\pi\tau/T)}e^{jn\omega_0 t}$$

Given:



then

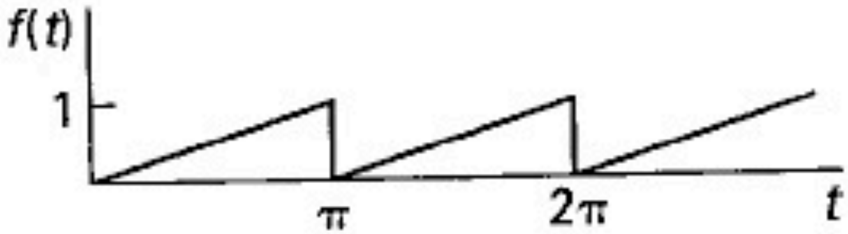
$$f_3(t) = \sum_{n=-\infty}^{\infty}A\delta(t - nT)$$

$$f_3(t) = (A/T) + (2A/T)\sum_{n=1}^{\infty}\cos(n\omega_0 t)$$

$$f_3(t) = (A/T)\sum_{n=-\infty}^{\infty}e^{jn\omega_0 t}$$

Fourier Series

What are the first terms in the Fourier series of the repeating function shown?



(A) $\frac{1}{2} - \cos 2t - \frac{1}{2}\cos 4t - \frac{1}{3}\cos 6t$

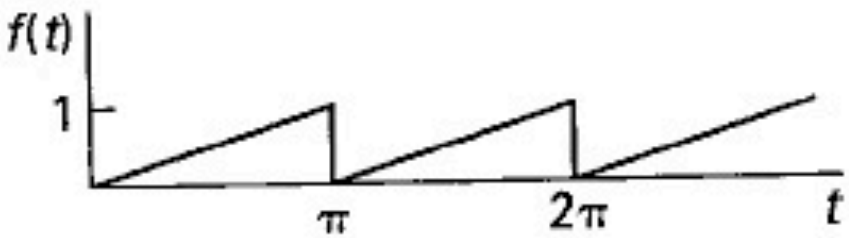
(B) $\frac{1}{2} - \frac{1}{\pi}\sin 2t - \frac{1}{2\pi}\sin 4t - \frac{1}{3\pi}\sin 6t$

(C) $\frac{1}{4} - \frac{1}{\pi}\left(\begin{array}{l}\cos 2t + \sin 2t + \cos 4t \\ \quad + \frac{1}{2}\sin 4t + \cos 6t + \frac{1}{3}\sin 6t\end{array}\right)$

(D) $\frac{1}{4} - \frac{1}{\pi}\left(\begin{array}{l}\frac{1}{\pi}\cos 2t + \sin 2t \\ \quad + \frac{1}{2\pi}\cos 4t + \frac{1}{2}\sin 4t \\ \quad + \frac{1}{3\pi}\cos 6t + \frac{1}{3}\sin 6t\end{array}\right)$

Fourier Series

What are the first terms in the Fourier series of the repeating function shown?



(A) $\frac{1}{2} - \cos 2t - \frac{1}{2}\cos 4t - \frac{1}{3}\cos 6t$

(B) $\frac{1}{2} - \frac{1}{\pi}\sin 2t - \frac{1}{2\pi}\sin 4t - \frac{1}{3\pi}\sin 6t$

(C) $\frac{1}{4} - \frac{1}{\pi}\left(\cos 2t + \sin 2t + \cos 4t + \frac{1}{2}\sin 4t + \cos 6t + \frac{1}{3}\sin 6t\right)$

(D) $\frac{1}{4} - \frac{1}{\pi}\left(\frac{1}{\pi}\cos 2t + \sin 2t + \frac{1}{2\pi}\cos 4t + \frac{1}{2}\sin 4t + \frac{1}{3\pi}\cos 6t + \frac{1}{3}\sin 6t\right)$

Fourier Transform (pg. 27, 29)

FOURIER TRANSFORM

The Fourier transform pair, one form of which is

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$$

$$f(t) = [1/(2\pi)] \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega$$

can be used to characterize a broad class of signal models in terms of their frequency or spectral content. Some useful transform pairs are:

$f(t)$	$F(\omega)$
$\delta(t)$	1
$u(t)$	$\pi\delta(\omega) + 1/j\omega$
$u\left(t + \frac{\tau}{2}\right) - u\left(t - \frac{\tau}{2}\right) = r_{rect}\,\frac{t}{\tau}$	$\tau \frac{\sin(\omega\tau/2)}{\omega\tau/2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$

Some mathematical liberties are required to obtain the second and fourth form. Other Fourier transforms are derivable from the Laplace transform by replacing s with $j\omega$ provided

$$f(t) = 0, t < 0$$

$$\int_0^{\infty} |f(t)|\, dt < \infty$$

The Fourier Transform and its Inverse

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

We say that $x(t)$ and $X(f)$ form a *Fourier transform pair*:

$x(t) \leftrightarrow X(f)$

Fourier Transform Pairs

$x(t)$	$X(f)$
1	$\delta(f)$
$\delta(t)$	1
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\Pi(t/\tau)$	$\tau\,\mathrm{sinc}\,(\tau f)$
$\mathrm{sinc}(Bt)$	$\frac{1}{B}\Pi(f/B)$
$\Lambda(t/\tau)$	$\tau\,\mathrm{sinc}^2\,(\tau f)$
$e^{-at}u(t)$	$\frac{1}{a + j2\pi f}$ $\quad a > 0$
$te^{-at}u(t)$	$\frac{1}{(a + j2\pi f)^2}$ $\quad a > 0$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2 + (2\pi f)^2}$ $\quad a > 0$
$e^{-(at)^2}$	$\frac{\sqrt{\pi}}{a} e^{-(\pi f/a)^2}$
$\cos(2\pi f_0 t + \theta)$	$\frac{1}{2}[e^{j\theta}\delta(f - f_0) + e^{-j\theta}\delta(f + f_0)]$
$\sin(2\pi f_0 t + \theta)$	$\frac{1}{2j}[e^{j\theta}\delta(f - f_0) - e^{-j\theta}\delta(f + f_0)]$
$\sum_{n=-\infty}^{n=+\infty} \delta(t - nT_s)$	$f_s \sum_{k=-\infty}^{k=+\infty} \delta(f - kf_s) \quad f_s = \frac{1}{T_s}$

Fourier Transform Theorems

Linearity	$ax(t) + by(t)$	$aX(f) + bY(f)$
Scale change	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Time reversal	$x(-t)$	$X(-f)$
Duality	$X(t)$	$x(-f)$
Time shift	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
Frequency shift	$x(t)e^{j2\pi f_0 t}$	$X(f - f_0)$
Modulation	$x(t)\cos 2\pi f_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
Multiplication	$x(t)y(t)$	$X(f) * Y(f)$
Convolution	$x(t) * y(t)$	$X(f)Y(f)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$

Fourier Transform

The Fourier transform of an impulse $a^2\delta(t)$ of magnitude a^2 is equal to

(A) $\sqrt{a}$

(B) $a - 1$

(C) a

(D) a^2

Fourier Transform

The Fourier transform of an impulse $a^2\delta(t)$ of magnitude a^2 is equal to

(A) $\sqrt{a}$

(B) $a - 1$

(C) a

(D) a^2

Laplace Transform (pg. 30)

LAPLACE TRANSFORMS

The unilateral Laplace transform pair

$$F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} dt$$

where $s = \sigma + j\omega$

represents a powerful tool for the transient and frequency response of linear time invariant systems. Some useful Laplace transform pairs are:

$f(t)$	$F(s)$
$\delta(t)$, Impulse at $t = 0$	1
$u(t)$, Step at $t = 0$	$1/s$
$t[u(t)]$, Ramp at $t = 0$	$1/s^2$
$e^{-\alpha t}$	$1/(s + \alpha)$
$te^{-\alpha t}$	$1/(s + \alpha)^2$
$e^{-\alpha t} \sin \beta t$	$\beta/[(s + \alpha)^2 + \beta^2]$
$e^{-\alpha t} \cos \beta t$	$(s + \alpha)/[(s + \alpha)^2 + \beta^2]$
$\dfrac{d^n f(t)}{dt^n}$	$s^n F(s) - \sum_{m=0}^{n-1} s^{n-m-1} \dfrac{d^m f(0)}{dt^m}$
$\int_0^t f(\tau) d\tau$	$(1/s)F(s)$
$\int_0^t x(t - \tau) h(\tau) d\tau$	$H(s)X(s)$
$f(t - \tau)\, u(t - \tau)$	$e^{-\tau s} F(s)$
$\lim_{t \to \infty} f(t)$	$\lim_{s \to 0} sF(s)$
$\lim_{t \to 0} f(t)$	$\lim_{s \to \infty} sF(s)$

The last two transforms represent the Final Value Theorem (F.V.T.) and Initial Value Theorem (I.V.T.), respectively. It is assumed that the limits exist.

Laplace Transform

What is the Laplace transform of $f(t) = e^{-6t}$?

(A) $\dfrac{1}{s+6}$

(B) $\dfrac{1}{s-6}$

(C) e^{-6+s}

(D) e^{6+s}

Laplace Transform

What is the Laplace transform of $f(t) = e^{-6t}$?

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(B) $\dfrac{1}{s-6}$

(C) e^{-6+s}

(D) e^{6+s}

Laplace Transform

What is the Laplace transform of the step function $f(t)$?

$$f(t) = u(t-1) + u(t-2)$$

(A) $\frac{1}{s} + \frac{2}{s}$

(B) $\frac{e^{-s} + e^{-2s}}{s}$

(C) $1 + \frac{e^{-2s}}{s}$

(D) $\frac{e^{s}}{s} + \frac{e^{2s}}{s}$

Laplace Transform

What is the Laplace transform of the step function $f(t)$?

$$f(t) = u(t-1) + u(t-2)$$

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(D) $\frac{e^{s}}{s} + \frac{e^{2s}}{s}$

Linear Algebra & Vectors

- Matrix Arithmetic
- Matrix Transpose and Inverse
- Determinant of a Matrix
- Solving Systems of Linear Equations
- Vector Addition and Subtraction
- Vector Dot and Cross Products
- Vector Identities
- Gradient, Divergence, and Curl

Matrix Arithmetic (pg. 30)

A matrix is an ordered rectangular array of numbers with *m* rows and *n* columns. The element a_{ij} refers to row *i* and column *j*.

Multiplication of Two Matrices

$$A = \begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix}$$ $A_{3,2}$ is a 3-row, 2-column matrix

$$B = \begin{bmatrix} H & I \\ J & K \end{bmatrix}$$ $B_{2,2}$ is a 2-row, 2-column matrix

In order for multiplication to be possible, the number of columns in A must equal the number of rows in B.

Multiplying matrix B by matrix A occurs as follows:

$$C = \begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix} \cdot \begin{bmatrix} H & I \\ J & K \end{bmatrix}$$

$$C = \begin{bmatrix} (A \cdot H + B \cdot J) & (A \cdot I + B \cdot K) \\ (C \cdot H + D \cdot J) & (C \cdot I + D \cdot K) \\ (E \cdot H + F \cdot J) & (E \cdot I + F \cdot K) \end{bmatrix}$$

Matrix multiplication is not commutative.

Addition

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} + \begin{bmatrix} G & H & I \\ J & K & L \end{bmatrix} = \begin{bmatrix} A+G & B+H & C+I \\ D+J & E+K & F+L \end{bmatrix}$$

Matrix Arithmetic

What is the matrix product **AB** of matrices **A** and **B**?

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

(A) $\begin{bmatrix} 10 & 4 \\ 7 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 11 & 4 \\ 5 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & 3 \\ 2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 10 & 7 \\ 4 & 3 \end{bmatrix}$

Matrix Arithmetic

What is the matrix product **AB** of matrices **A** and **B**?

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(C) $\begin{bmatrix} 8 & 3 \\ 2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 10 & 7 \\ 4 & 3 \end{bmatrix}$

Matrix Transpose and Inverse (pg. 30)

Identity Matrix

The matrix $\mathbf{I} = (a_{ij})$ is a square $n \times n$ matrix with 1's on the diagonal and 0's everywhere else.

Matrix Transpose

Rows become columns. Columns become rows.

$$A = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \quad A^T = \begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

Inverse $[\ ]^{-1}$

The inverse $\boldsymbol{B}$ of a square $n \times n$ matrix $\boldsymbol{A}$ is

$$\boldsymbol{B} = \boldsymbol{A}^{-1} = \frac{\text{adj}(\boldsymbol{A})}{|\boldsymbol{A}|}, \text{ where}$$

adj($\boldsymbol{A}$) = adjoint of $\boldsymbol{A}$ (obtained by replacing $\boldsymbol{A}^T$ elements with their cofactors) and $|\boldsymbol{A}|$= determinant of $\boldsymbol{A}$.

$[\mathbf{A}][\mathbf{A}]^{-1} = [\mathbf{A}]^{-1}[\mathbf{A}] = [\mathbf{I}]$ where $\mathbf{I}$ is the identity matrix.

Matrix Transpose and Inverse

What is the transpose of matrix **A**?

$$\mathbf{A} = \begin{bmatrix} 5 & 8 & 5 & 8 \\ 8 & 7 & 6 & 2 \end{bmatrix}$$

(A) $\begin{bmatrix} 8 & 7 & 6 & 2 \\ 5 & 8 & 5 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 6 & 7 & 8 \\ 8 & 5 & 8 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & 5 \\ 7 & 8 \\ 6 & 5 \\ 2 & 8 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 8 \\ 8 & 7 \\ 5 & 6 \\ 8 & 2 \end{bmatrix}$

Matrix Transpose and Inverse

Using the property that $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$ for two square matrices, what is $|\mathbf{A}^{-1}|$ in terms of $|\mathbf{A}|$ for any invertible square matrix $\mathbf{A}$?

(A) $\dfrac{1}{|\mathbf{A}|}$

(B) $\dfrac{1}{|\mathbf{A}^{-1}|}$

(C) $\dfrac{|\mathbf{A}|}{|\mathbf{A}^{-1}|}$

(D) $\dfrac{|\mathbf{A}^{-1}|}{|\mathbf{A}|}$

Matrix Transpose and Inverse

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(B) $\dfrac{1}{|\mathbf{A}^{-1}|}$

(C) $\dfrac{|\mathbf{A}|}{|\mathbf{A}^{-1}|}$

(D) $\dfrac{|\mathbf{A}^{-1}|}{|\mathbf{A}|}$

Matrix Transpose and Inverse

The cofactor matrix of matrix **A** is **C**.

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 6 & -8 & -1 \\ -5 & 10 & 0 \\ -2 & 1 & 2 \end{bmatrix}$$

What is the inverse of matrix **A**?

(A) $\begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.50 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$

(B) $\begin{bmatrix} 0.25 & 0.50 & 0.33 \\ 0.33 & 0.50 & 0.50 \\ 0.50 & 1.0 & 0.25 \end{bmatrix}$

(C) $\begin{bmatrix} 1.2 & -1.0 & -0.40 \\ -1.6 & 2.0 & 0.20 \\ -0.20 & 0 & 0.40 \end{bmatrix}$

(D) $\begin{bmatrix} 0.80 & 0.40 & -0.60 \\ 0.20 & -0.40 & 0.40 \\ -0.40 & 0.60 & 0.80 \end{bmatrix}$

Matrix Transpose and Inverse

The cofactor matrix of matrix $\mathbf{A}$ is $\mathbf{C}$.

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 6 & -8 & -1 \\ -5 & 10 & 0 \\ -2 & 1 & 2 \end{bmatrix}$$

What is the inverse of matrix $\mathbf{A}$?

(A) $\begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.50 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$

(B) $\begin{bmatrix} 0.25 & 0.50 & 0.33 \\ 0.33 & 0.50 & 0.50 \\ 0.50 & 1.0 & 0.25 \end{bmatrix}$

(C) $\begin{bmatrix} 1.2 & -1.0 & -0.40 \\ -1.6 & 2.0 & 0.20 \\ -0.20 & 0 & 0.40 \end{bmatrix}$

(D) $\begin{bmatrix} 0.80 & 0.40 & -0.60 \\ 0.20 & -0.40 & 0.40 \\ -0.40 & 0.60 & 0.80 \end{bmatrix}$

Determinants (pg. 31)

A *determinant of order n* consists of n^2 numbers, called the *elements* of the determinant, arranged in n rows and n columns and enclosed by two vertical lines.

In any determinant, the *minor* of a given element is the determinant that remains after all of the elements are struck out that lie in the same row and in the same column as the given element. Consider an element which lies in the jth column and the ith row. The *cofactor* of this element is the value of the minor of the element (if $i + j$ is *even*), and it is the negative of the value of the minor of the element (if $i + j$ is *odd*).

If n is greater than 1, the *value* of a determinant of order n is the sum of the n products formed by multiplying each element of some specified row (or column) by its cofactor. This sum is called the *expansion of the determinant* [according to the elements of the specified row (or column)]. For a second-order determinant:

$$\begin{vmatrix} a_1 a_2 \\ b_1 b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

For a third-order determinant:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants

What is the determinant of matrix **A**?

$$\mathbf{A} = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

(A) 0

(B) 15

(C) 14

(D) 26

Determinants

What is the determinant of matrix **A**?

$$\mathbf{A} = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

(A) 0

(B) 15

(C) 14

(D) 26

Determinants

For the following set of equations, what is the determinant of the coefficient matrix?

$$10x + 3y + 10z = 5$$
$$8x - 2y + 9z = 5$$
$$8x + y - 10z = 5$$

(A) 598

(B) 620

(C) 714

(D) 806

Determinants

For the following set of equations, what is the determinant of the coefficient matrix?

$$10x + 3y + 10z = 5$$
$$8x - 2y + 9z = 5$$
$$8x + y - 10z = 5$$

(A) 598

(B) 620

(C) 714

(D) 806

Systems of Linear Equations

Using Cramer's rule, what values of x, y, and z will satisfy the following system of simultaneous equations?

$$2x + 3y - 4z = 1$$

$$3x - y - 2z = 4$$

$$4x - 7y - 6z = -7$$

(A) $x = 1, \ y = -4, \ z = -1$

(B) $x = 1, \ y = 3, \ z = 1$

(C) $x = 3, \ y = -2, \ z = 4$

(D) $x = 3, \ y = 1, \ z = 2$

Systems of Linear Equations

Using Cramer's rule, what values of x, y, and z will satisfy the following system of simultaneous equations?

$$2x + 3y - 4z = 1$$

$$3x - y - 2z = 4$$

$$4x - 7y - 6z = -7$$

(A) $x = 1,\ y = -4,\ z = -1$

(B) $x = 1,\ y = 3,\ z = 1$

(C) $x = 3,\ y = -2,\ z = 4$

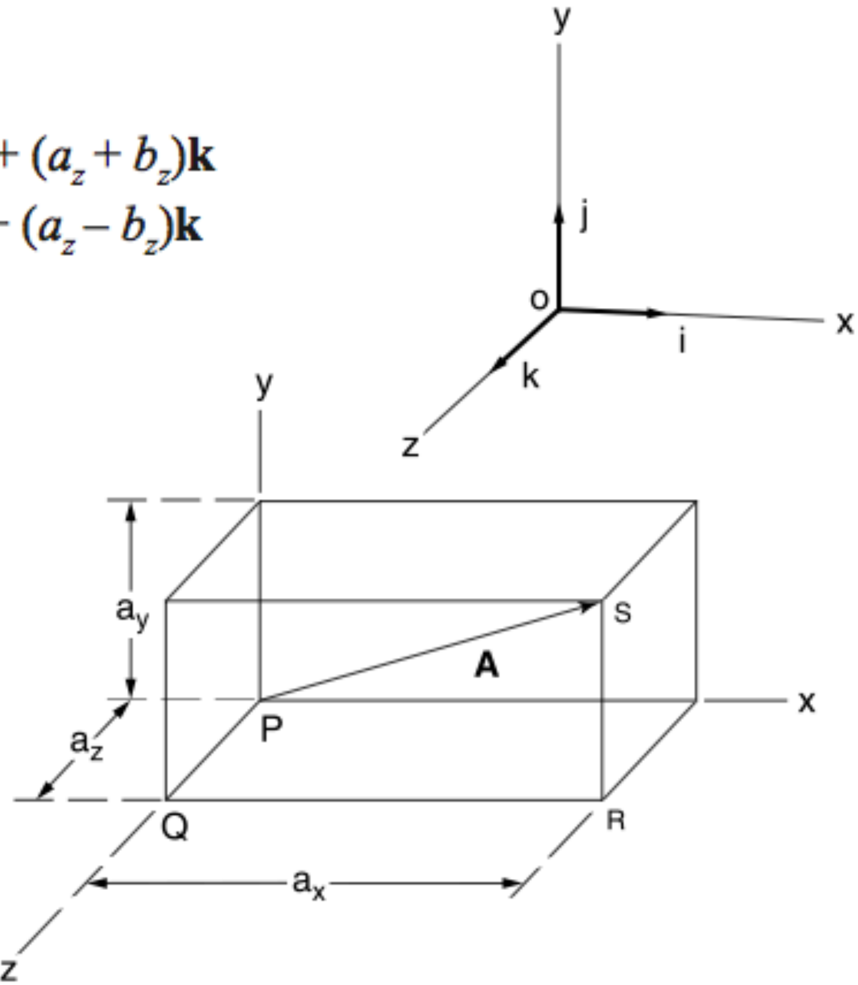
(D) $x = 3,\ y = 1,\ z = 2$

Vector Addition and Subtraction (pg. 31)

Addition and *subtraction:*

$$\mathbf{A} + \mathbf{B} = (a_x + b_x)\mathbf{i} + (a_y + b_y)\mathbf{j} + (a_z + b_z)\mathbf{k}$$

$$\mathbf{A} - \mathbf{B} = (a_x - b_x)\mathbf{i} + (a_y - b_y)\mathbf{j} + (a_z - b_z)\mathbf{k}$$



$$\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

Vector Addition and Subtraction

Find the unit vector (i.e., the direction vector) associated with the origin-based vector $18\mathbf{i} + 3\mathbf{j} + 29\mathbf{k}$.

(A) $0.525\mathbf{i} + 0.088\mathbf{j} + 0.846\mathbf{k}$

(B) $0.892\mathbf{i} + 0.178\mathbf{j} + 0.416\mathbf{k}$

(C) $1.342\mathbf{i} + 0.868\mathbf{j} + 2.437\mathbf{k}$

(D) $6\mathbf{i} + \mathbf{j} + \frac{29}{3}\mathbf{k}$

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Vector Addition and Subtraction

What is the sum of the two vectors $5\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ and $10\mathbf{i} - 12\mathbf{j} + 5\mathbf{k}$?

(A) $8\mathbf{i} - 7\mathbf{j} - \mathbf{k}$

(B) $10\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$

(C) $15\mathbf{i} - 9\mathbf{j} - 2\mathbf{k}$

(D) $15\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$

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(D) $15\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$

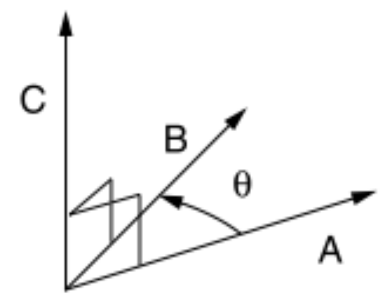
Vector Dot and Cross Products (pg. 31)

The *dot product* is a *scalar product* and represents the projection of **B** onto **A** times |**A**|. It is given by

$$\mathbf{A} \bullet \mathbf{B} = a_x b_x + a_y b_y + a_z b_z$$
$$= |\mathbf{A}||\mathbf{B}| \cos\theta = \mathbf{B} \bullet \mathbf{A}$$

The *cross product* is a *vector product* of magnitude |**B**| |**A**| sin θ which is perpendicular to the plane containing **A** and **B**. The product is

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = -\,\mathbf{B} \times \mathbf{A}$$



The sense of **A** × **B** is determined by the right-hand rule.

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}|\mathbf{n} \sin\theta, \text{ where}$$

n = unit vector perpendicular to the plane of **A** and **B**.

Vector Dot and Cross Products

What is the dot product, $\mathbf{A}\cdot\mathbf{B}$, of the vectors $\mathbf{A} = 2\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$ and $\mathbf{B} = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$?

(A) $-4\mathbf{i} + 4\mathbf{j} - 32\mathbf{k}$

(B) $-4\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$

(C) -40

(D) -32

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Vector Dot and Cross Products

What is the cross product, $\mathbf{A} \times \mathbf{B}$, of vectors $\mathbf{A}$ and $\mathbf{B}$?

$$\mathbf{A} = \mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$
$$\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$$

(A) $\mathbf{i} - \mathbf{j} - \mathbf{k}$

(B) $-\mathbf{i} + \mathbf{j} + \mathbf{k}$

(C) $2\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$

(D) $2\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

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Vector Identities (pg. 31)

$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$; $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

$\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2$

$\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$

$\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$

If $\mathbf{A} \cdot \mathbf{B} = 0$, then either $\mathbf{A} = 0$, $\mathbf{B} = 0$, or $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C})$

$(\mathbf{B} + \mathbf{C}) \times \mathbf{A} = (\mathbf{B} \times \mathbf{A}) + (\mathbf{C} \times \mathbf{A})$

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$

$\mathbf{i} \times \mathbf{j} = \mathbf{k} = -\mathbf{j} \times \mathbf{i}$; $\mathbf{j} \times \mathbf{k} = \mathbf{i} = -\mathbf{k} \times \mathbf{j}$

$\mathbf{k} \times \mathbf{i} = \mathbf{j} = -\mathbf{i} \times \mathbf{k}$

If $\mathbf{A} \times \mathbf{B} = \mathbf{0}$, then either $\mathbf{A} = \mathbf{0}$, $\mathbf{B} = \mathbf{0}$, or $\mathbf{A}$ is parallel to $\mathbf{B}$.

$\nabla^2 \phi = \nabla \cdot (\nabla \phi) = (\nabla \cdot \nabla) \phi$

$\nabla \times \nabla \phi = \mathbf{0}$

$\nabla \cdot (\nabla \times \mathbf{A}) = \mathbf{0}$

$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

Vector Identities

What is the dot product $\mathbf{A}\cdot\mathbf{B}$ of unit vectors $\mathbf{A} = 3\mathbf{i}$ and $\mathbf{B} = 2\mathbf{i}$?

(A) -6

(B) -5

(C) 5

(D) 6

Vector Identities

What is the dot product $\mathbf{A}\cdot\mathbf{B}$ of unit vectors $\mathbf{A} = 3\mathbf{i}$ and $\mathbf{B} = 2\mathbf{i}$?

(A) –6

(B) –5

(C) 5

(D) 6

Gradient, Divergence, and Curl (pg. 31)

Gradient, Divergence, and Curl

$$\nabla \phi = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right)\phi$$

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right) \cdot (V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k})$$

$$\nabla \times \mathbf{V} = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right) \times (V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k})$$

The Laplacian of a scalar function ϕ is

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

Gradient, Divergence, and Curl

What is the divergence of the following vector field?

$$\mathbf{V} = 2x\mathbf{i} + 2y\mathbf{j}$$

(A) 0

(B) 2

(C) 3

(D) 4

Gradient, Divergence, and Curl

What is the divergence of the following vector field?

$$\mathbf{V} = 2x\mathbf{i} + 2y\mathbf{j}$$

(A) 0

(B) 2

(C) 3

(D) 4

Gradient, Divergence, and Curl

Determine the curl of the vector function $\mathbf{V}(x, y, z)$.

$$\mathbf{V}(x, y, z) = 3x^2\mathbf{i} + 7e^x y\mathbf{j}$$

(A) $7e^x y$

(B) $7e^x y\mathbf{i}$

(C) $7e^x y\mathbf{j}$

(D) $7e^x y\mathbf{k}$

Gradient, Divergence, and Curl

Determine the curl of the vector function $\mathbf{V}(x, y, z)$.

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(B) $7e^x y\mathbf{i}$

(C) $7e^x y\mathbf{j}$

(D) $7e^x y\mathbf{k}$

Gradient, Divergence, and Curl

Determine the Laplacian of the scalar function $\frac{1}{3}x^3 - 9y + 5$ at the point $(3, 2, 7)$.

(A) 0

(B) 1

(C) 6

(D) 9

Gradient, Divergence, and Curl

Determine the Laplacian of the scalar function $\frac{1}{3}x^3 - 9y + 5$ at the point $(3, 2, 7)$.

(A) 0

(B) 1

(C) 6

(D) 9